NAME: 2014-02-03

Each task is worth 10 points.

1. Let $G = (W \cup U, E)$ where $W = \{w_1, w_2, \ldots, w_n)\}$, $U = \{u_1, u_2, \ldots u_n\}$ and $E = \{\{w_i, u_j\} : 1 \le i, j \le n\}$. Calculate the number of Hamiltonian cycles in G.

2. Prove that for every graph $G = (V,E)$ the relation R

*x*R*y iff there exists an x-y path in G*

is an equivalence relation on V.

3. Determine whether the following proposition is a tautology: $((p \Rightarrow q) \wedge (r \Rightarrow s)) \Rightarrow ((p \vee r) \Rightarrow (q \vee s))$.

4. Find $\bigcup_{t \in N} A_t$ and $\bigcap_{t \in N} A_t$ where $A_t = \{(x,y) \in R^2 : |y| > tx\}$ and N is the set of natural numbers (including 0).

5. We are seating 5 married couples around a table (the seats are identical). Let $\{M_1, M_2, M_3, M_4, M_5\}$ be the set of men and let $\{W_1, W_2, W_3, W_4, W_5\}$ be the set of their wives. In how many cases the man 1 will be seated next to his wife and the man 3 will not seat next to his wife?

6. Consider a 5×5 array. In how many ways can we fill the array with X-s and O-s so that no two consecutive rows are identical?