NAME:

- Each task is worth 10 points. 1. Prove, by induction on n = |X|, the Cantor's formula: $2^{X/2} = 2^{|X|}$ for every finite set X.
- 2. Consider graph G=(V,E) where V= $\{0,1,\ldots,7\}$ and $\{i,j\} \in E$ iff (i-j) mod $3 \neq 0$.
 - (a) Is G Hamiltonian? If it is, find a Hamiltonian cycle. Otherwise prove that it is not.
 - (b) Is G Eulerian? If it is, find an Eulerian cycle. Otherwise prove that it is not.
- 3. Determine if the following equality is true for every three sets A, B, C: $A \cup (B \setminus C) = [(A \cup B) \setminus C] \cup (A \cap B)$. If not, find an example of A, B, C, for which it is not true.
- 4. Find $\bigcup_{t \in N} A_t$ and $\bigcap_{t \in N} A_t$ where $A_t = \{x \in \mathbb{R}: 3 + (-1)^t \frac{(-1)^t}{t} < x < 7 + (-1)^t \frac{(-1)^t}{t}\}$ and N is the set of natural numbers (without 0).
- 5. We are seating 9 different people in a row. Three of them are Chinese, three are Polish and the remaining three are Russian. In how many cases there will be no two people of the same nationality sitting next to each other?
- 6. In how many ways can we distribute 7 identical roses and 5 different cookies among 4 ladies so that no lady gets more than 3 roses?