FUNCTIONS

Problem 1. Which of the following relations are functions:
(a) aRb iff $a^2=b^2$, $R\subseteq \mathbf{R}\times\mathbf{R}$,
(b) aRb iff $a^2=b^2$, $R\subseteq \mathbf{N}\times\mathbf{N}$,
(c) aRb iff $a^2=b^2$, $R\subseteq \mathbf{R}\times\mathbf{R}^+$,
(d) aRb iff Re a = Im b, $R\subseteq \mathbf{C}\times\mathbf{C}$,
(e) Divisibility from **N** into **N**.
Problem 2. $f:X\to Y$ is "onto" and $g:Y\to Z$ is such, that $g\circ f$ is 1-1. Show that g is 1-1. Show that if f is not onto then g may not me 1-1.
Problem 3. Find an example of a function $f:X\to X$ which is onto but not 1-1.
Problem 4. Find an example of a function $f:X\to X$ which is 1-1 but not onto.
Problem 5. Show that if X is finite then every 1-1 function $f:X\to X$ is onto and vice versa.
Problem 6. $f:\mathbf{N}^2\to\mathbf{N}$ is defined as follows: $f((n,k))=n+k+1$.
(a) is f onto?
(b) is f 1-1?
(c) find $f(\mathbf{N}\times\{1\})$
(d) find $f^{-1}(\{0\})$
(e) find $f^{-1}(\{2n:n\in\mathbf{N}\})$.
Problem 7. $f:X\to Y$, A,B are subsets of X; S,T are subsets of Y. Show that
(a) $f(A\cup B)=f(A)\cup f(B)$
(b) $f(A\cap B)\subseteq f(A)\cap f(B)$; show that one cannot replace "$\subseteq$" with "=",
(c) $A\subseteq B$ implies $f(A)\subseteq f(B)$,
(d) $f^{-1}(S\cup T)= f^{-1}(S)\cup f^{-1}(T)$,
(e) $f^{-1}(S\cap T)= f^{-1}(S)\cap f^{-1}(T)$,
(f) $S\subseteq T$ implies $f^{-1}(S)\subseteq f^{-1}(T)$.