INDUCTION

Problem 1. Prove that $\sum_{i=1}^{n} i(i!) = (n+1)! - 1$.

Problem 2. Prove that $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$.

Problem 3. Prove that using only three-cent and five-cent stamps one can make *n* cents postage for every $n \geq 8$.

Problem 4. Prove that for any positive integer *n*, $2^{2n}-1$ is divisible by 3.

Problem 5. Prove by induction the Newton's binomial formula $(a+b)^n = \sum_{i=0}^{n} \binom{n}{i} a^{n-i} b^i$.

Problem 6. Prove that the number of k-element subsets of an n-element set is $\binom{n}{k}$.

Problem 7. Prove that for any $n \geq 5$, $n^2 > 4n+1$.

Problem 8. Prove that for any $n \geq 5$, $2^n > n^2$.

Problem 9. Prove by induction that for any positive integer *k*, the product of *k* consecutive integers is a multiple of *k!*

Problem 10. Prove that for every associative algebra (X,*), for every *n* and for every $x_1, x_2, \ldots x_n$

$$(\ldots((x_1 * x_2) * x_3) * \ldots) * x_n = x_1 * (\ldots * (x_{n-2} * (x_{n-1} * x_n))\ldots).$$

Problem 11. Prove the pigeonhole principle.

Problem 12. Prove that for every commutative and associative algebra (X,*), for every *n*, for every $x_1, x_2, \ldots x_n$ and for every permutation $\pi: X \rightarrow X$

$$x_1 * x_2 * x_3 * \ldots * x_n = x_{\pi(1)} * x_{\pi(2)} * x_{\pi(3)} * \ldots * x_{\pi(n)}$$

Problem 13. Prove by induction that the number of all functions from a *k*-element set into an *n*-element set is n^k