Partial Orders

1. Describe all maximal chains in (**N**,|).
2. Prove that the largest element in a poset X is the only maximal element in X.
3. Prove that the least element in a poset X is the only minimal element in X
4. Suppose that x_0 is the only minimal (maximal) element in a poset X. Does this imply that x_0 is the least (largest) element in X?
5. For every positive integer *n* find an example of a poset with precisely *n* minimal (maximal) elements.
6. Can an element of a poset be at the same time minimal and maximal?
7. Is there an ordering relation such that every element of the poset is both maximal and minimal? If YES, describe all such relations.
8. Prove that if (X,R) is a poset then, for every $Y \subseteq X$, $(Y, R \cap (Y \times Y))$; is a poset.
9. Is there an ordering relation, which is at the same time an equivalence relation? If YES, describe all such relations.
10. Let $X=\{2^n : n \in \{1,2, \ldots\}\} \cup \{3\}$ and let R be the divisibility relation on X. Find all maximal and all minimal elements in the poset (X,R).
11. Prove that in every finite poset there exists at least one minimal element and at least one maximal element.
12. Is it true that in a finite poset the only maximal element is the largest element (the only minimal element is the least element)?
13. Let R and S be partial orders on X.
    a. is $R \cap S$ a partial order on X?
    b. is $R \cup S$ a partial order on X?
    c. is $R \times S$ a partial order on $X \times X$?
14. Show that in a totally ordered set, every finite subset has the largest element.