

Relations, Equivalence relations

1. For each of the given relations on $\{1,2,3,6\}$ list all its elements and sketch its graph.
 - a. xRy iff x divides y (in symbols: $x|y$),
 - b. xRy iff $x < y$
 - c. xRy iff $x = y$
 - d. xRy iff $x \neq y$
 - e. xRy iff $x^2 \leq y$
 - f. xRy iff $x+y$ is odd
2. Determine which of the following relations on $\mathbb{N} \setminus \{1\}$ are equivalence relations. Find equivalence classes for those who are.
 - a. xRy iff $\gcd(x,y) = 1$ (\gcd stands for “greatest common divisor”)
 - b. xRy iff $\gcd(x,y) > 1$
 - c. xRy iff $\text{lcm}(x,y) = xy$ (guess what lcm stands for)
 - d. xRy iff x divides y
 - e. xRy iff $x+y$ is even
 - f. xRy iff $\gcd(x,y)$ divides $\text{lcm}(x,y)$
 - g. xRy iff $\text{lcm}(x,y)$ divides $\gcd(x,y)$
 - h. xRy iff $x+y$ is odd
 - i. xRy iff xy is odd
3. Find all numbers k such that the relation R_k defined on the set of natural numbers as: pR_kq if and only if $k/(p+q)$ is an equivalence relation. For each k for which the answer is YES find the equivalence classes of R_k .
4. R_1 and R_2 are equivalence relations on a set X . In each of the following questions, if the answer is YES, describe equivalence classes of the relation in terms of equivalence classes of R_1 and R_2 .
 - a. Is $R_1 \cap R_2$ an equivalence relation on X ?
 - b. Is $R_1 \dot{\cup} R_2$ an equivalence relation on X ?
 - c. Is $R_1 \setminus R_2$ an equivalence relation on X ?
 - d. Is $R_1 \dot{\cup} R_2$ an equivalence relation on $X \times X$?
5. Show that for every partition p of a set X there exists an equivalence relation on X whose equivalence classes are exactly elements of p .
6. Let X denote the set of all 0-1 sequences of length 100. We define the following relation R on X :
 $aRb \hat{=} |\{i : a(i)=b(i)\}|$ is even
 Is R an equivalence on X ?
 If it is, describe its equivalence classes.