EXAM 1 2017-JUNE-21

1. Find the number of n-long sequences of letters a, b, c, where letter 'a' cannot appear on odd numbered positions. Positions are numbered starting from 1
2. G is a graph whose vertices have degrees 3 and 5 only and vertices of the same degree are not adjacent. Find the average degree of a vertex in G.
3. (Tricky one) Prove or disprove:
   a. for every graph G, if (1,2,2,2,3,4,4,5) is the sequence of degrees of vertices of G then G is connected.
   b. for every graph G, if (1,2,2,2,3,4,4,5) is the sequence of degrees of vertices of G then G is disconnected.
4. Let R and S be equivalence relations on a set X. Prove that $R\cap S$ is an equivalence relation on X. Describe equivalence classes of $R\cap S$ in terms of equivalence classes of R and S.
5. Find the sets $\bigcap_{t\in R^+} A_t$ and $\bigcup_{t\in R^+} A_t$, where $A_t = \{x \in R : 9\frac{x^2}{t^2} - 1 = 0\}$.

EXAM 2 2017-JUNE-30

1. How many sequences each consisting of all letters of the word "PATAGONIA" don't have identical letters next to each other?
2. Let $n\in\mathbf{N}$. Find the number of solutions of the equation $x_1+x_2+x_3+x_4 = n$ where $x_1,x_2,x_3,x_4\in\mathbf{N}$ and $x_2+x_4\geq 3$. **N** denotes the set on natural numbers, including zero.
3. Continue the following definitions using ONLY mathematical and logical symbols (do not use descriptive terms like *dim, lim, span, exp* etc.):
   a) dimension of V (a vector space over a field F) is n if and only if …
   b) $\chi(G) = k$ (chromatic number of a graph G is k) if and only if …
4. Prove by induction :
   $(\forall n \in N)[n > 7 \Rightarrow (\exists k_1 \in \{0,1,2,...\})(\exists k_2 \in \{0,1,2,...\})\, n = 3k_1 + 5k_2]$.
5. % is a relation on the set of all subsets of **N** (the set of natural numbers), A%B iff A÷B is a finite set. Determine if % is an equivalence relation (A÷B denotes the symmetric difference, i.e. A÷B = (A∪B)-(A∩B)).

EXAM 3 (RETAKE) 2017-SEP-14

1. Prove that the operation of symmetric difference on sets is associative
2. In how many ways can you put 10 identical gold coins into four colored boxes so that at least 1 goes into the blue box, at least 1 into yellow, at most 2 into red and at least 3 into green?
3. Vertices of the graph G are binary sequences of length 99, two vertices (sequences) are adjacent iff they differ on an odd number of positions (e.g. (0,0, … , 0) is adjacent to (1,1,1,0, … ,0), but not to to (1,1,0,0, … ,0) etc.). Is G Eulerian?
4. Consider the following relation @ on the set of all equivalence relations on a set X: R@S iff every equivalence class of R is a subset of some equivalence class of S. Is @ a partial order?
5. Complete the statements below using only mathematical and logical symbols (no words).

   (a) A graph G=(V,E) is connected iff

   (b) n mod $k = r$ iff