EXAM 1 2018-JAN-29

1. We have unlimited supply of balloons in n colors. Calculate the number of ways to give 2 differently colored balloons to each of k people if
   a. no 2 people can get the same pair of colors
   b. no 2 people can get the same color.
2. Find the number of permutations of the set A={1, ..., n}, where numbers divisible by 5 are not next to each other.
3. Complete the following definitions using ONLY mathematical and logical symbols. For example
   A set $S=\{v_1,v_2,\dots,v_n\}$ is a basis for a vector space V over a field **F** iff
   $((\forall a_1,a_2,\dots,a_n\in\mathbf{K})(a_1v_1+a_2v_2+\dots+a_nv_n=\Theta \Rightarrow a_1=a_2=\dots=a_n=0))$ & $((\forall w\in V)(\exists b_1,b_2,\dots,b_n\in\mathbf{K})\ w = b_1v_1+b_2v_2+\dots+b_nv_n)$

   a) $n \bmod k = p$ iff
   b) $\lim_{n\to\infty} a_n = L$ iff
   c) $(X,\ll)$ is a poset. An element p is a *maximal* element of $(X,\ll)$ iff
4. G is a graph whose vertex set is {1,2, … 100} and whose edge set is $\{\{i,j\}: 0<|i-j|<5\}$.
   (a) Is G connected? (b) Is G Eulerian (c) Is G planar?
5. $\cong$ is a relation on the set $\mathbf{R}\times\mathbf{R}$ such that $(x,y)\cong(p,q)$ iff $py=xq$. Verify if it is an equivalence relation. If it is, describe its equivalence classes.

EXAM 2 2018-FEB-02

1. G=(V,E) is a graph such that V={1,2, … ,n} and $E=\{\{i,j\}: |i-j|=1 \vee |i-j|=n-1\}$. Calculate the number of bijections $f : V \to V$ such for every $p,q\in V$ if p is adjacent to q then f(p) or f(q) is even.
2. Find the number of sequences consisting of all letters of the word "NOSOLUTIONS" where every two consecutive letters are different..
3. Complete the following definitions using ONLY mathematical and logical symbols. For example:
   W=span(S) **F** iff $(\forall w\in W)(\exists n\in\mathbf{N})(\exists b_1,b_2,\dots,b_n\in\mathbf{K})(\exists v_1,v_2,\dots,v_n\in S)\ w = b_1v_1+b_2v_2+\dots+b_nv_n$.
   a) a natural number p is a prime iff (do not use the divisibility symbol |)
   b) R is an equivalence relation on a set X iff
   c) a subset t of the Cartesian product X×Y is an "onto" function iff
4. G is a graph whose vertex set is {1,2, … , 82}, vertices i and j are adjacent iff $(i-j) \bmod 4 = 0$ and $i\neq j$.
   (a) Calculate the chromatic number of G (b) Is G Eulerian? (c) Is G planar?
5. Prove that the operation of symmetric difference of sets is associative, i.e. for every A,B,C – subsets of the universal set X $A\div(B\div C) = (A\div B)\div C$.

COMMITTEE EXAM 2018-FEB-14

1. How many numbers n, $1\le n\le 1000000$ have an odd sum of their digits (in decimal notation)?
2. In how many ways can you load 27 identical rounds of ammo into 4 identical magazines, each of capacity 13, so that at least 5 rounds go into every magazine?
3. (a) Define a connected graph using only mathematical and logical symbols (no words allowed).
   (b) Prove that every graph with p vertices whose every vertex has degree at least p/2 is connected.
4. (a) Quote the definition of a partial order, i.e. complete the statement "a set X is partially ordered by a relation R if and only if …" using only logical and set-theory symbols.
   (b) Assuming that (X,R) is a poset write (using formal symbols only) that (X,R) has a maximal element.
5. Recall that a relation on a set X is a set of ordered pairs of element of X. What is the intersection of the family of all equivalence relations on the set of natural numbers **N**? *Hint: it is a relation on N but what relation?*