ETMAG, Exam 1, 18.06.2013

Question1	Question2	Question3	Question4	Question5	Sum	Exercises	Total

Name:
Index number:

Question 1. Consider a function

$$f(x) = \begin{cases} x^3 + 4x^2 + 4x & x < -2 \\ 0 & x = -2 \\ (x+2)\sin\frac{1}{x+2} & x > -2. \end{cases}$$

1. Is the function $y = f(x)$ is continuous at -2?
2. Calculate if possible $f'_-(-2)$.
3. Calculate if possible $f'_+(-2)$.
4. Does $f'(-2)$ exist?
5. Find all asymptotes of $f(x)$.

Question 2. Find all eigenvalues and eigenvectors of the following matrix over $\mathbb{R}$. For each eigenspace find its basis and dimension.

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & -2 & 2 \end{pmatrix}$$

Name:
Index number:

Question 3. (10pts) (a) Calculate, if possible, the following limits or show they fail to exist:

$$\lim_{x\to\infty} x\sin x^2, \qquad \lim_{n\to\infty} \frac{1}{n^2+1}+\frac{1}{n^2+2}+\ldots+\frac{1}{n^2+n}.$$

(10pts) (b) Let A be a square matrix over a field $\mathbb{K}$ and λ_1, λ_2 be its two different eigenvalues. Assume that v_1 is a non-zero eigenvector for λ_1 and v_2 is a non-zero eigenvector for λ_2. Prove that the set $\{v_1, v_2\}$ is a linearly independent set.

Question 4. (5pts) (a) Write an equation of the line tangent to $f(x) = \ln(x^2 + e)$ at $x_0 = 0$.
(15pts) (b) Determine monotonicity and find the extreme values of the following function:

$$g(x) = \sqrt{12 \cdot x^2 - x^3}$$

Name:
Index number:

Question 5. (10pts) (a) Find polar form of z^2, $-\bar{z}$, and $(z+\bar{z})^2$ if $z = \cos\alpha + i\sin\alpha$.
(10pts) (b) Calculate

$$\frac{(1-\sqrt{3}i)^{100}}{(-1-i)^{200}} \qquad \sqrt[3]{-32}.$$

Notes