

Question1	Question2	Question3	Question4	Question5	Sum	Exercises	Total

$(G \times H, \circ)$ , where  $(G, \cdot)$  and  $(H, \otimes)$  are groups and  $\circ$  is coordinate-wise product, i.e.

$$(a, b) \circ (c, d) = (a \cdot c, b \otimes d) \text{ for } (a, b), (c, d) \in G \times H.$$

**Name:**

**Index number:**

**Question 1.** Calculate, if possible, the following limits:

(a) (8pts)

$$\lim_{x \rightarrow \infty} \frac{x^2 + \sin x}{3x^2 - 2x + \cos x}, \quad \lim_{n \rightarrow \infty} \left( \frac{n-1}{n-2} \right)^{n+5}$$

(4pts) (b) A long rectangular sheet of metal, 12 inches wide, is to be made into a symmetric rain gutter by turning up two sides at angles of 45 to the sheet. How many inches should be turned up to give the gutter its greatest capacity.

**Question 2.** Let  $f(x) = x \cdot \sqrt{8 - x^2}$ . Determine the domain of  $f(x)$  and intervals on which the function is monotonic. Find its extreme values. Sketch the graph of  $f(x)$ .

**Question 3.** Find all eigenvalues and eigenvectors of the following matrix over  $\mathbb{R}$ . For each eigenspace find its basis and dimension.

$$\begin{pmatrix} 0 & 0 & 2 \\ 0 & -1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

**Question 4.** (6pts) (a) Calculate

$$\sqrt[4]{(15 + 23i)^4} \quad \frac{(1 - i)^{200}}{(-\sqrt{3} + i)^{100}} \quad \sqrt[4]{-1}$$

(6pts)(b) Knowing that  $1 + 3i$  is a root of  $160 - 32x + 26x^2 - 2x^3 + x^4$  find all remaining roots.

**Question 5.** (6pts) (a) Find all asymptotes of  $h(x)$ :

$$h(x) = \frac{25(x^3 + 2x^2 + 3x + 4)}{5x^2 + 6x + 7}$$

(b)(6pts) Consider a sequence  $a_n$  recursively defined as follows:

$$a_1 = 0 \quad a_n = \sqrt{6 + a_{n-1}} \text{ for } n > 1$$

Show that  $a_n$  is convergent and find its limit.