Question1	Question2	Question3	Question4	Question5	Sum	Exercises	Total

Name:
Index number:

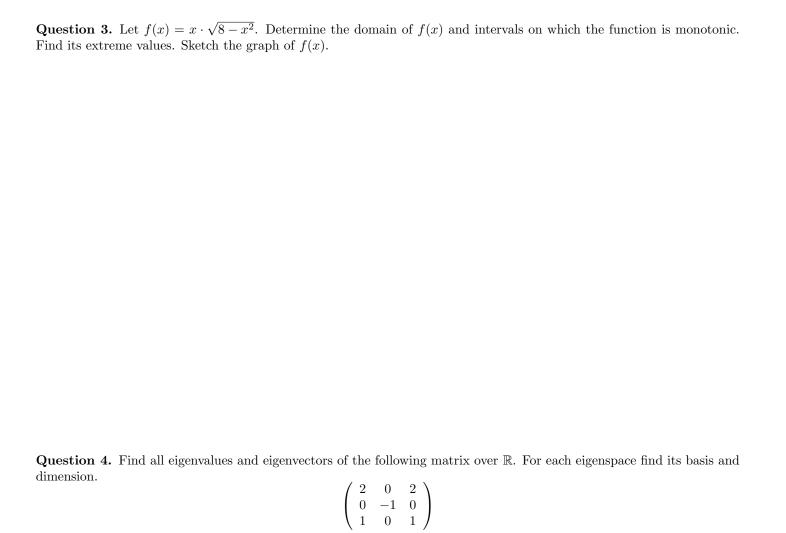**Question 1.** (6pts) (a) Calculate $\sqrt[4]{(15+23i)^4}$, $\frac{(1-i)^{200}}{(-\sqrt{3}+i)^{100}}$ and $\sqrt[4]{-1}$. (6pts)(b) Knowing that $1-3i$ is a root of

$$160 - 32x + 26x^2 - 2x^3 + x^4$$

find all remaining roots.

Question 2. Calculate, if possible, the following limits:

(a) (8pts)

$$\lim_{x \to \infty} \frac{x^2 + \sin x}{3x^2 - 2x + \cos x}, \qquad \lim_{n \to \infty} \left(\frac{n-1}{n-2} \right)^{n+8}$$

(4pts) (b) A long rectangular sheet of metal, 12 inches wide, is to be made into a symmetric rain gutter by turning up two sides at angles of 45 to the sheet. How many inches should be turned up to give the gutter its greatest capacity.

Question 3. Let $f(x) = x \cdot \sqrt{8 - x^2}$. Determine the domain of $f(x)$ and intervals on which the function is monotonic. Find its extreme values. Sketch the graph of $f(x)$.

Question 4. Find all eigenvalues and eigenvectors of the following matrix over $\mathbb{R}$. For each eigenspace find its basis and dimension.

$$\begin{pmatrix} 2 & 0 & 2 \\ 0 & -1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

Question 5. (6pts) (a) Find all asymptotes of $h(x)$:

$$h(x) = \frac{25(x^3 + 2x^2 + 3x + 4)}{5x^2 + 6x + 7}$$

(b)(6pts) Consider a sequence a_n recusively defined as follows: $a_1 = 0$ and $a_n = \sqrt{6 + a_{n-1}}$ for $n > 1$. Show that a_n is convergent.