ETMAG, Exam 1, 5.02.2015

Question1	Question2	Question3	Question4	Question5	Sum	Exercises	Total

Name: **Index number:**

Question 1. (a) Find parameters a and b for which the following function is continuous

$$f(x) = \begin{cases} (2a+b)\cdot\frac{x}{x+\sin x} & x<0, \\ 2 & x=0, \\ a+\frac{e^x-1}{x} & x>0 \end{cases}$$

(b) Check *differentiability* of the function $y = g(x)$ at $x_0 = 1$ if $g(x) = |x-1|\cdot(x-1)+x$.

Question 2. Find all eigenvalues and eigenvectors of the following matrix over $\mathbb{R}$. For each eigenspace find its basis and dimension.

$$\begin{pmatrix} 3 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 0 & 3 & 1 & 0 \\ 0 & 5 & 2 & 3 \end{pmatrix}$$

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Question 3. (a) Determine the monotonicity and convexity of the function $f(x)$. Find its extreme values.

$$f(x) = (x-1)^2 \cdot e^{2x}$$

(b) Calculate the limits:

$$\text{(a)} \lim_{n\to\infty} \sqrt[n]{n + 3^n + 7^n + \arctan(2015 \cdot n)} \qquad \text{(b)} \lim_{x\to\infty} \frac{5x^2 + \sin x}{2x^2 + \cos x}.$$

Question 4. (a) Let V and W be vector spaces over a field $\mathbb{K}$. Which of the following mappings $F : V \to W$ are linear:

1. $F((x, y)) = (y, x + 3y)$ for $V = W = \mathbb{R}^2$ over $\mathbb{R}$
2. $F(x, y, z) = (2x + x^2, x + y, z)$ for $V = W = \mathbb{R}^3$, over $\mathbb{R}$.

(b) Find a formula and the matrix for the linear operator $F : \mathbb{R}^3 \to \mathbb{R}^3$ such that:

$$F(1, 1, 1) = (0, 0, 1) \text{ and } F(1, 1, 0) = F(0, 1, 1) = (0, 1, 1)$$

Name: **Index number:**

Question 5. (a) Calculate

$$\sqrt[6]{1} \qquad \left(\frac{1-i}{1+i}\right)^{100}$$

(b) Find all $z \in \mathbb{C}$ satisfying $\mathfrak{Re}(z \cdot (1+i)) + z \cdot \bar{z} = 0$.

(c) You run a canoe-rental business on a small river in Ohio. You currently charge 12 per canoe and average 36 rentals a day. An industry journal says that, for every fifty-cent increase in rental price, the average business can expect to lose two rentals a day. Use this information to attempt to maximize your income. What should you charge?

Notes