ETMAG, Exam 2, 30.06.2014

Questi	on1	Question2	Question3	Question4	Question5	Sum	Exercises	Total

Name: Index number:

Question 1. Let $f(x) = x \cdot \sqrt{8 - x^2}$. Determine the domain of f(x) and intervals on which the function is monotonic. Find its extreme values. Sketch the graph of f(x). **Question 2.** Calculate, if possible, the following limits: (a) (8pts)

$$\lim_{x \to \infty} \frac{x^2 + 2\sin x}{3x^2 - 2x - \cos x}, \qquad \qquad \lim_{n \to \infty} \left(\frac{n+1}{n-2}\right)^{n+5}$$

(4pts) (b) A long rectangular sheet of metal, 12 inches wide, is to be made into a symmetric rain gutter by turning up two sides at angles of 45 to the sheet. How many inches should be turned up to give the gutter its greatest capacity.

Name: Index number:

Question 3. Find all eigenvalues and eigenvectors of the following matrix over \mathbb{R} . For each eigenspace find its basis and dimension.

$$\left(\begin{array}{rrr} 0 & 0 & 2 \\ 0 & -1 & 0 \\ 1 & 0 & 1 \end{array}\right)$$

Question 4. (10pts) (a) Calculate

$$\sqrt[4]{(13+61\cdot i)^4}$$
 $\frac{(1+i)^{200}}{(-\sqrt{3}+i)^{100}}$ $\sqrt[4]{-1}$

(10 pts)(b) Knowing that 1 - 3i is a root of $160 - 32x + 26x^2 - 2x^3 + x^4$ find all remaining roots.

Name: Index number:

Question 5. (10pts) (a) Find all asymptotes of h(x):

$$h(x) = \frac{25(x^3 + 5x^2 + 4x + 3)}{5x^2 + 2x + 1}$$

(b)(10pts) Consider a sequence a_n recusively defined as follows: $a_1 = 0$ $a_n = \sqrt{6 + a_{n-1}}$ for n > 1. Show that a_n is convergent and find its limit.

Notes