ETMAG, Exam 2, 12.02.2014

Question1	Question2	Question3	Question4	Question5	Sum	Exercises	Total

Name: **Index number:**

Question 1. (a) Find parameters a and b for which the following function is continuous

$$f(x) = \begin{cases} (2a+b) \cdot x^2 \cdot \frac{1}{e^{x^2}-1} & x < 0, \\ 2 & x = 0, \\ a + \ln x \cdot \sin x & x > 0 \end{cases}$$

(b) Check *differentiability* of the function $y = g(x)$ at $x_0 = 5$ if $g(x) = |x+5| \cdot (x+5)^2$.

Question 2. Find all eigenvalues and eigenvectors of the following matrix over $\mathbb{R}$. For each eigenspace find its basis and dimension.

$$\begin{pmatrix} 3 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 0 & 3 & 1 & 0 \\ 0 & 5 & 2 & 3 \end{pmatrix}$$

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Question 3. (a) Determine the monotonicity and convexity of the function $f(x)$. Find its extreme values.

$$f(x) = (x-1)^2 \cdot e^{2x}$$

(b) Calculate the limits:

$$\text{(a)} \lim_{n\to\infty} \sqrt[n]{2^n + 3^n + \sin(2014 \cdot n)} \qquad \text{(b)} \lim_{n\to\infty} \left(\frac{2n+3}{2n-5}\right)^{n-3}.$$

Question 4. (a) Let V and W be vector spaces over field $\mathbb{K}$. Which of the following mappings $F : V \to W$ are linear:

1. $F((x, y)) = (x + y, x - 2y)$ for $V = W = \mathbb{R}^2$ over $\mathbb{R}$
2. $F(x, y, z) = (2x + y, x \cdot z, 0)$ for $V = W = \mathbb{R}^3$, over $\mathbb{R}$.

(b) Find a formula and the matrix for the linear operator F such that:

$$F(1, 1, 1) = (1, 0, 1), F(1, 1, 0) = F(0, 1, 1) = (1, 1, 1)$$

Name: **Index number:**

Question 5. (a) Calculate

$$\sqrt[6]{1} \qquad \left(\frac{1-i}{1+i}\right)^{100}$$

(b) Find all $z \in \mathbb{C}$ satisfying $\mathfrak{Re}(z \cdot (1-i)) + z \cdot \bar{z} = 0$.

(c) A rectangular box with square base and top is to be made to contain 1250 cubic meters. The material for the base cost 35 cents per square meter, for the top 15 cents per square meter, and for the sides 20 cents per square meter. Find the dimensions that will minimize the cost of the box.

Notes