ETMAG, Exam 2, 11.02.2015

Question1	Question2	Question3	Question4	Question5	Sum	Exercises	Total

Name: Index number:

Question 1. (a) Calculate $\sqrt[4]{(15+23i)^4}$, $\frac{(1-i)^{200}}{(-\sqrt{3}+i)^{100}}$.

 $A \twoheadrightarrow B$

(b) Knowing that 1 - 3i is a root of $160 - 32x + 26x^2 - 2x^3 + x^4$ find all remaining roots.

Question 2. (a) Calculate, if possible, the following limits:

$$\lim_{x \to \infty} x \arctan\left((-1)^{[x]} \cdot x \right), \qquad \qquad \lim_{n \to \infty} \left(\frac{n-1}{n-2} \right)^{n+8}$$

(b) Find two nonnegative numbers whose sum is 9 and so that the product of one number and the square of the other number is a maximum.

Question 3. Let $f(x) = x \cdot \sqrt{8 - x^2}$. Determine the domain of f(x) and intervals on which the function is monotonic. Find its extreme values. Sketch the graph of f(x). Question 4. Find all eigenvalues and eigenvectors of the following matrix over \mathbb{R} . For each eigenspace find its basis and dimension.

$$\left(\begin{array}{rrrr} 2 & 0 & 2 \\ 0 & -1 & 0 \\ 1 & 0 & 1 \end{array}\right)$$

Question 5. (6pts) (a) Find all asymptotes of h(x):

$$h(x) = \frac{25(x^3 + 2x^2 + 3x + 4)}{5x^2 + 6x + 7}$$

(b)(6pts) Consider a sequence a_n recusively defined as follows: $a_1 = 0$ and $a_n = \sqrt{6 + a_{n-1}}$ for n > 1. Show that a_n is convergent.