ETMAG, Exam 3, 10.09.2013

Question1	Question2	Question3	Question4	Question5	Sum	Exercises	Total

Name:
Index number:

Question 1. (a) Give definition of a continuous function at a given point. Show that if two functions $y = f(x)$ and $y = g(x)$ are continuous at point x_0 then the function $y = f(x) + g(x)$ is also continuous.
(b) Check *differentiability* of the following function at $x_0 = -1$:

$$g(x) = |x + 1| \cdot e^{x+1}$$

Question 2. Find all eigenvalues and eigenvectors of the following matrix over $\mathbb{R}$. For each eigenspace find its basis and dimension.

$$\begin{pmatrix} 3 & 0 & 0 \\ 2 & 2 & 0 \\ 1 & 1 & 2 \end{pmatrix}$$

Name:
Index number:

Question 3. (a) Find equation of the tangent line to the graph of the function $y = f(x)$ at x_0 if

$$f(x) = \sqrt[x]{\frac{1}{x}} \text{ and } x_0 = 1.$$

(b) Show that $e^x > 1 + x$ for $x \neq 0$.

Question 4. (a) Verify linear independence of the following sets of vectors:

- $\{(1,0,1),(1,1,0),(0,1,1)\}$ in $\mathbb{Z}_2^3$ over $\mathbb{Z}_2$,
- $\{1, \sin x, e^x\}$ in $\mathbb{R}^{\mathbb{R}}$ over $\mathbb{R}$.

(b) Find a formula and the matrix for the linear operator F such that:

$$F(1,0,1) = (1,0,1), F(1,1,0) = F(0,1,1) = (1,1,1)$$

Name:
Index number:

Question 5. (a) Calculate

$$\sqrt[8]{-1} \qquad\qquad \left(\frac{1+i}{1-i}\right)^{200}$$

(b) Find all $z \in \mathbb{C}$ satisfying:

$$\mathfrak{Re}(z \cdot (2-i)) + z \cdot \bar{z} = 0$$

Notes