

Question1	Question2	Question3	Question4	Question5	Sum	Exercises	Total

Name:**Index number:****Question 1.** (10pts) (a) Check differentiability of the following function:

$$f(x) = \begin{cases} (x+1) \cdot e^{-\frac{1}{|x+1|}} & x \neq -1 \\ 0 & x = -1 \end{cases}$$

(10pts)(b) A rectangular enclosure is to be constructed so that it will have one side along an existing wall and the other three sides fenced. There are 1000 m of fence available. What is the largest possible area for the enclosure?

Question 2. Find all eigenvalues and eigenvectors of the following matrix over \mathbb{R} . For each eigenspace find its basis and dimension.

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & -2 & 2 \end{pmatrix}$$

Question 3. (10pts) (a) Calculate, if possible, the following limits or show they fail to exist:

$$\lim_{x \rightarrow \infty} x \sin x^2, \quad \lim_{n \rightarrow \infty} \frac{1}{n^2 + 1} + \frac{1}{n^2 + 2} + \dots + \frac{1}{n^2 + n}.$$

(10pts) (b) Show that if for two $n \times n$ matrices A, B over the field of real numbers there is an invertible $n \times n$ matrix P such that

$$A = P^{-1} \cdot B \cdot P$$

then A and B have the same eigenvalues.**Question 4.** (5pts) (a) Write an equation of the line tangent to $f(x) = \ln(x^2 + e)$ at $x_0 = 0$.

(15pts) (b) Determine monotonicity and find the extreme values of the following function:

$$g(x) = \sqrt{12 \cdot x^2 - x^3}$$

Question 5. (10pts) (a) Find polar form of z^2 , $-\bar{z}$, and $(z + \bar{z})^2$ if $z = \cos \alpha + i \sin \alpha$.

(10pts) (b) Calculate

$$\frac{(1 - \sqrt{3}i)^{100}}{(-1 - i)^{200}} \quad \sqrt[3]{-32}.$$