

JORDAN BLOCK MATRICES – EXERCISES WITH ANSWERS

Find the Jordan block matrix J similar to the matrix A and a change-of-basis matrix P such that $J=P^{-1}AP$. Verify!

$$1. A = \begin{bmatrix} -1 & -2 & -3 & 2 \\ 4 & 4 & 5 & -3 \\ 4 & 3 & 6 & -3 \\ 6 & 3 & 8 & -3 \end{bmatrix}, J = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$2*. A = \begin{bmatrix} -1 & -5 & 1 & 4 \\ -2 & -4 & 2 & 4 \\ 1 & -1 & -1 & 0 \\ -3 & -6 & 3 & 6 \end{bmatrix}, J = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$3. A = \begin{bmatrix} -2 & 1 & 1 & 1 \\ -1 & -1 & 1 & 1 \\ 1 & 1 & -2 & -1 \\ -2 & 0 & 2 & 1 \end{bmatrix}, J = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$4. A = \begin{bmatrix} 2 & 1 & 0 & 1 & -2 \\ 2 & 5 & -6 & 1 & 1 \\ 2 & 4 & -4 & 1 & 0 \\ 0 & 2 & -2 & 0 & 1 \\ 5 & 8 & -10 & 3 & 0 \end{bmatrix}, J = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$5*. A = \begin{bmatrix} 0 & -1 & -2 & 1 \\ 2 & 2 & 3 & -1 \\ 2 & 0 & 5 & -1 \\ 2 & -2 & 5 & 1 \end{bmatrix}, J = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

$$6. A = \begin{bmatrix} 21 & 18 & -53 & -31 \\ 8 & 7 & -20 & -11 \\ 13 & 10 & -32 & -19 \\ -4 & -2 & 9 & 6 \end{bmatrix}, J = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$7. A = \begin{bmatrix} 2 & -3 & -4 & 6 \\ 2 & -4 & -7 & 10 \\ 0 & -2 & -1 & 4 \\ 1 & -4 & -5 & 9 \end{bmatrix}, J = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

$$8. A = \begin{bmatrix} -2 & -3 & 3 & 5 \\ -2 & 1 & 2 & 2 \\ 0 & -1 & 1 & 1 \\ -4 & -2 & 4 & 6 \end{bmatrix}, J = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

$$9. A = \begin{bmatrix} 7 & 2 & -4 & -2 \\ 4 & 1 & -2 & -2 \\ 12 & 4 & -7 & -4 \\ 4 & 0 & -2 & -1 \end{bmatrix}, J = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$10. A = \begin{bmatrix} 9 & 2 & -4 & -2 \\ -7 & 0 & 3 & 2 \\ 0 & 0 & 1 & 0 \\ 17 & 5 & -9 & -3 \end{bmatrix}, J = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

$$11. A = \begin{bmatrix} 7 & 2 & -3 & -1 \\ -7 & -2 & 5 & 1 \\ -1 & -1 & 3 & 0 \\ 9 & 3 & -5 & 0 \end{bmatrix}, J = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

$$12. A = \begin{bmatrix} -2 & 0 & 0 & 0 \\ 8 & 6 & -8 & 0 \\ 4 & 4 & -6 & 0 \\ 4 & 4 & -4 & -2 \end{bmatrix}, J = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$