NAME 10 FEB 2015

MiNI - ALGEBRA - FINAL EXAM - time allowed 120 minutes.

Do not talk, cheat or else...!

1. Find any linear mapping $T:\mathbf{R}^4 \to \mathbf{R}^4$ such that Im(T)=span{(1,1,0,0),(0,0,1,1)}.
2. Prove that if F is a one-to-one linear mapping and $\{v_1, v_2, \ldots, v_k\}$ is linearly independent then $\{F(v_1), F(v_2), \ldots, F(v_k)\}$ is also linearly independent.
3. Find the diagonal matrix for F(x,y,z) = (-13x+11y-13z, 14x-10y+13z, 28x-22y+27).
4. For the matrices J and A from the previous problem, find P such that $J=P^{-1}AP$.
5. Calculate $\operatorname{Re}\left(\frac{\sqrt{3}-1+(\sqrt{3}+1)i}{2-2i}\right)^{1000}$ and $\operatorname{Im}\left(\frac{\sqrt{3}-1+(\sqrt{3}+1)i}{2-2i}\right)^{1000}$. Hint: *Represent the numerator as a sum of two numbers of equal moduli and use geometry to find its argument*
6. Prove that a group (G,∇) is commutative if and only if for each $a,b\in G$, $a^2\nabla b^2 = (a\nabla b)^2$. ($x^2 = x\nabla x$)