LAST NAME…………………….………………….FIRST NAME……………………..………2015.09.11

ALGEBRA RETAKE EXAM Time allowed 120 min. Each task is worth 12 points.

1. Find the polar form of

(a) $z = \cos\alpha - i\sin\alpha$ (b) $z = \sin\alpha + i\cos\alpha$

(c) $\dfrac{(1-i\sqrt{3})^{100}}{(-1-i)^{200}}$ (d) $z = \sqrt[3]{-64}$ (each of them)

2. Determine which of the following sets are linearly independent in the indicated vector spaces. Explain.

(a) $\{x^4+x^2+x,\ x^3+x^2,\ x^4-x^3+x\}$ in $\mathbf{R}[x]$ over $\mathbf{R}$

(b) $\{\sin x, \cos x, \cos 2x\}$ in $\mathbf{R}^{\mathbf{R}}$ over $\mathbf{R}$.

3. Find dimensions of the following vector spaces. Justify your answers.

(a) The space of all those polynomials from $\mathbf{R}_6[x]$ (*i.e. of degree at most 7*), who have roots at 1 and -1.

(b) $\{(x,y,z,t)\in\mathbf{R}^4 : x+y = z+t\}$.

4. $A = \begin{bmatrix} 5 & 6 & 3 & -6 \\ 0 & 2 & 0 & 0 \\ -6 & -6 & -4 & 6 \\ 0 & 3 & 0 & -1 \end{bmatrix}$ find a diagonal matrix B similar to A.