

1.  $F$  is a linear operator such that  $F(3,2,2) = (3,1,2)$ ,  $F(1,2,1) = (2,2,3)$  and  $F(2,3,2) = (5,-1,0)$ . If possible, find  $\text{kernel}(F)$ . If not possible, explain why.
2. Prove that the set  $G$  of all complex roots of 1, of all possible orders is an Abelian group with respect to multiplication. (  $G = \{z \in \mathbb{C} : (\exists n \in \mathbb{N}) z^n = 1\}$  )
3.  $J = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$   $A = \begin{bmatrix} 3 & -1 & -3 \\ -1 & 3 & 3 \\ 1 & -1 & -1 \end{bmatrix}$ . Find a matrix  $P$  such that  $J = P^{-1}AP$ . Hint: *it exists!*
4. Consider  $\mathbb{C}$  a vector space over  $\mathbb{R}$ . Verify that  $f(z) = \bar{z}$  is a linear operator. Find eigenvalues of  $f$ , for each eigenvalue find an eigenvector.
5. Solve in complex numbers the equation  $z^4 = (-3 + 5i)^4$