LAST NAME……………………..…………….FIRST NAME……………………..………2016.02.04

ALGEBRA FINAL EXAM Time allowed 120 min. Each task is worth 12 points.

1. Find the polar form of

(a) $z = \cos\alpha - i\sin\alpha$ (b) $\dfrac{(1-i\sqrt{3})^{100}}{(-1-i)^{200}}$ (c) $z = \sqrt[3]{-64}$ (each of them)

2. Determine which of the following sets are linearly independent in the indicated vector spaces. Explain.

(a) $\{x^4+x^2+x,\ x^3+x^2,\ x^4-x^3+x\}$ in $\mathbf{R}[x]$ over $\mathbf{R}$

(b) $\{\sin x, \cos^2 x, \cos 2x\}$ in $\mathbf{R}^{\mathbf{R}}$ over **R.**

3. Find dimensions of the following vector spaces. Justify your answers.

(a) The space of all those polynomials from $\mathbf{R}_6[x]$ (*i.e. of degree at most 6*), who have roots at 1 and -1.

(b) $\{(x,y,z,t)\in\mathbf{R}^4 : x+2y+3z+4t=0 \ \&\ x+y+z+t=0\}$.

4. $F(x,y,z,t) = (5x+6y+3z-6t\ ,\ 2y\ ,\ -6x-6y-4z+6t\ ,\ 3y-t)$. Find a basis S in $\mathbf{R}^4$ such that $M_S(F)$ is diagonal. Find also the matrix itself.

5. Prove that if A and B are matrices of the same linear operator then det A = det B. *Hint*: det XY= detX detY