

LAST NAME.....FIRST NAME.....8 Sept 2016

ALGEBRA RETAKE EXAM

Time allowed 120 min.

Each task is worth 12 points.

1. Find the polar form of

(a)  $z = -\cos \alpha - i \sin \alpha$  (b)  $\frac{(1-i\sqrt{3})^{100}}{(-1+i)^{200}}$  (c)  $z = \sqrt[3]{-64}$  (each of them)

2. Determine which of the following sets are linearly independent in the indicated vector spaces. Explain.

(a)  $\{x^4+2x^2+x, x^3+x^2, x^4-x^3+x\}$  in  $\mathbf{R}[x]$  over  $\mathbf{R}$

(b)  $\{1, \sin x, \cos^2 x, \cos 2x\}$  in  $\mathbf{R}^{\mathbf{R}}$  over  $\mathbf{R}$ .

3. Find dimensions of the following vector spaces. Justify your answers.

(a) The space of all those polynomials from  $\mathbf{R}_6[x]$  (i.e. of degree at most 6), who have roots at 1 and 0.

(b)  $\{(x,y,z,t) \in \mathbf{R}^4 : x+2y+3z+4t=0 \text{ \& } x+y+z+t=0\}$ .

4.  $F(x,y,z,t) = (5x+6y+3z-6t, 2y, -6x-6y-4z+6t, 3y-t)$ . Find a basis S in  $\mathbf{R}^4$  such that  $M_S(F)$  is diagonal. Find also the matrix itself.

5. Prove that if A and B are matrices of the same linear operator then  $\det A = \det B$ . *Hint:  $\det XY = \det X \det Y$*