NAME 07 FEB 2017

MiNI - ALGEBRA - FINAL EXAM 2 - time allowed 100 minutes.
Do not talk, cheat or else...!

1. (a) Quote the definition of the matrix of a linear operator $F:V\to V$ with respect to two bases, R and S of V.
   (b) $F: R^3\to R^3$ scales every vector by 2. Find its matrix in bases R={(1,1,1), (1,1,0), (1,0,0)} and S={(0,1,1), (1,0,1), (1,1,0)}.
2. (a) Quote the definition of a group.
   (b) Verify if the set G of all complex solutions to equations of the form $z^n=1$, for n=3,4, … is a group with respect to multiplication. (To be more precise: $G=\{z\in C:(\exists n>2)z^n=1\}$)
3. $J=\begin{bmatrix}2&0&0\\0&2&0\\0&0&1\end{bmatrix}$ $A=\begin{bmatrix}3&-1&-3\\-1&3&3\\1&-1&-1\end{bmatrix}$. Find a matrix P such that $J=P^{-1}AP$. Hint: *it exists!*
4. Consider C as a vector space over R. Verify that $f(z)=\bar{z}$ is a linear operator. Find all eigenvalues of f, for each eigenvalue find an eigenvector.
5. Solve in complex numbers the equation $z^3=\bar{z}$.