NAME ALG-MiNI_FINAL 2018-JAN-30

Time 120 min. Each task is 12 points. Include all necessary comments and calculations. SUBMIT THIS ONLY. Do not talk, cheat or else...!

1. Is (**R**,#, ∆) a field where, for every x,y from **R,** $x\#y = x+y+1$ and $x\Delta y = xy+x+y$?
2. Find all complex numbers satisfying $z^5 = \bar{z}$.
3. Show that span(v_1,v_2, ... ,v_n) = span(v_1,v_1+v_2,v_2+v_3, ... ,$v_{n-1}+v_n$).
4. Let $A=\begin{bmatrix} 1 & 1 & 2 & 1 \\ 3 & 1 & 2 & 0 \\ -1 & 2 & 3 & 1 \\ 2 & 4 & 3 & 2 \end{bmatrix}$ be the matrix of some linear operator F in a basis S={v_1, v_2, v_3, v_4}. Find the matrix of F in the basis R={w_1,w_2, w_3, w_4} where $w_1= v_3$, $w_2= v_4$, $w_3= v_2$ and $w_4= v_1$.
5. $A=\begin{bmatrix} 1 & -2 & 3 & 4 \\ 1 & 4 & -3 & -4 \\ 2 & 4 & -4 & -8 \\ -1 & -2 & 3 & 6 \end{bmatrix}$ is the matrix for some F in the standard basis S. Find, if possible, a basis R and a diagonal matrix D such that $D=M_R(F)$.