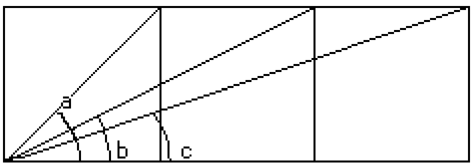
1. Find a linear mapping T:$\mathbf{R}^4 \rightarrow \mathbf{R}^4$ such that im(T)=span{(1,-1,2,1),(2,1,3,1)}. Represent your operator in the standard form T(x,y,z,t)= … .
   Since (1,0,0,0),(0,1,0,0),(0,0,1,0) and (0,0,0,1) span R^4, their images by T span im(T). So it is enough to put T(1,0,0,0)=(1,-1,2,1), T(0,1,0,0)=T(0,0,1,0)=T(0,0,0,1)=(2,1,3,1). This results in T(x,y,z,t) = x(1,-1,2,1) + y(2,1,3,1) + z(2,1,3,1) + t(2,1,3,1) = (x+2y+2z+2t,-x+y+z+t,2x+3y+3z+3t,x+y+z+t). Obviously that is only one of infinitely many.
2. Prove that if $A=M^S{}_R(F)$ and $B=M^R{}_S(F)$, then $A = B^{-1}$.
   To prove this we must demonstrate that $I=AB=M^S{}_R(F)M^R{}_S(F)$. Now, $M^S{}_R(F)M^R{}_S(F) = M^R{}_R(F\circ F)$ and there is no reason why $M^R{}_R(F\circ F)$ should be the identity matrix, except if F is the identity operator. Hence the statement we were asked to prove is false and as such not provable.
3. Find the Jordan block matrix J similar to the matrix $A=\begin{bmatrix} 0 & -3 & -1 & 1 \\ 2 & 5 & 1 & -1 \\ -1 & -1 & 2 & 0 \\ 1 & 2 & 1 & 1 \end{bmatrix}$.
   The characteristic polynomial for A is $(2-\lambda)^4$. The rank of A-2I is 2 and $(A-2I)^2=[0]$. Hence J has two blocks and it has 2 blocks of the size at least 2 each. So $J=\begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}$
4. Check with uncle Tom that your J matrix is right. Then find P such that $J=P^{-1}AP$.
   As mentioned above, $(A-2I)^2=[0]$, So all we need is to multiply A-λI by two non-eigenvectors and make sure that the resulting vectors form a basis. One matrix P is $\begin{bmatrix} -2 & 1 & -1 & 0 \\ 2 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$. Of course there are infinitely many others
5. Let F:V→W and G:W→T be linear mappings. Prove that G∘F is a linear mapping and ker(F)⊆ker(G∘F).
   (G∘F)(u+v)=G(F(u+v))=G(F(u)+F(v))=G(F(u))+G(F(v))=(G∘F)(u)+(G∘F)(v)
   (G∘F)(*p*v)=G(F(*p*v))=G(*p*F(v))=*p*G(F(v))=*p*(G∘F)(v), so G∘F preserves both addition and scaling, hence it is linear.
   If v∈ker(F) then F(v)=Θ. Then (G∘F)(v)=G(F(v))=G(Θ)=Θ, which means v∈ker(G∘F).
6. <u>Using complex numbers</u> show that $a+b+c=\frac{\pi}{2}$



Assuming sides of the squares are all equal to 1 and the bottom left corner is (0,0), the other ends of diagonals are (1,1), (2,1) and (3,1) or, in the complex numbers notation, 1+i, 2+i, 3+i. Obviously, (1+i)(2+i)(3+i)=|(1+i)(2+i)(3+i)|(cos(a+b+c)+isin(a+b+c)). Now (1+i)(2+i)(3+i) = (1+3i)(3+i) = 10i = $10(\cos(2k\pi+\frac{\pi}{2})+i\sin(2k\pi+\frac{\pi}{2}))$. Since $0<a+b+c<\pi$ we get $a+b+c=\frac{\pi}{2}$.