1. Calculate $\operatorname{Re}\left(\frac{\sqrt{2}+i\sqrt{6}}{2i-2}\right)^{666}$.

Solution. $\left(\frac{\sqrt{2}+i\sqrt{6}}{2i-2}\right)^{666} = \left(\frac{\sqrt{8}\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)}{\sqrt{8}\left(\frac{\sqrt{2}}{2}i-\frac{\sqrt{2}}{2}\right)}\right)^{666} = \left(\frac{\frac{1}{2}+i\frac{\sqrt{3}}{2}}{-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i}\right)^{666} = \left(\frac{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}{\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}}\right)^{666} =$

$\frac{\cos\frac{666\pi}{3}+i\sin\frac{666\pi}{3}}{\cos 666\frac{3\pi}{4}+i\sin 666\frac{3\pi}{4}} = \frac{\cos 0+i\sin 0}{\cos\frac{999\pi}{2}+i\sin\frac{999\pi}{2}} = \frac{\cos 0+i\sin 0}{\cos\left(500\pi-\frac{\pi}{2}\right)+i\sin\left(500\pi-\frac{\pi}{2}\right)} =$

$\frac{\cos 0+i\sin 0}{\cos\frac{-\pi}{2}+i\sin\frac{-\pi}{2}} = \cos\frac{\pi}{2}+i\sin\frac{\pi}{2}$. Finally, $\operatorname{Re}\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right) = \cos\frac{\pi}{2} = 0$

2. Let (F,#,&) be a field and let Δ be the identity element for #. Prove that for every x and y from F, if $x \neq \Delta$ and $y \neq \Delta$ then $x \& y \neq \Delta$.
   Solution. Let E denote the identity element for &, and let x^{-1} denote the inverse to x with respect to &. Suppose $x\&y = \Delta$. Then $x^{-1}(x\&y) = x^{-1}\Delta$, which implies y=Δ, contrary to our assumption.

3. Prove that $\mathbf{R}_2[\times]$ (the set of all polynomials of degree at most 2) is a vector space over the field of real numbers. Vector addition and scaling are regular operations on functions.
   Solution. $\mathbf{R}_2[x]$ is obviously nonempty, the sum of two polynomials from $\mathbf{R}_2[x]$ belongs to $\mathbf{R}_2[x]$ and a multiple of a polynomial from $\mathbf{R}_2[x]$ belongs to $\mathbf{R}_2[x]$. So $\mathbf{R}_2[x]$ is a subspace of the space of all functions from $\mathbf{R}$ into $\mathbf{R}$, so it is a vector space.

4. Find all complex numbers satisfying $z^7 = \bar{z}$.
   Solution. $z^7 = \bar{z}$ implies $z^8 = \bar{z}z = |z|^2$. Taking modulus of both sides we get $|z|^8 = |z|^2$, which implies $|z| = 0$ or $|z| = 1$. Hence we get $z^8 = 0$ or $z^8 = 1$. In the first case we have z=0. In the second case we just the root formula to find all 8 roots of 1 of order 8.