1. Calculate $\left(\frac{1+i\sqrt{3}}{2+2i}\right)^{246}$

Solution. $\left(\frac{1+i\sqrt{3}}{2+2i}\right)^{246} = \left(\frac{\frac{1}{2}+i\frac{\sqrt{3}}{2}}{\sqrt{2}\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}\right)^{246} = \frac{1}{2^{123}}\left(\frac{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}{\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}}\right)^{246}$

$= \frac{1}{2^{123}}\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)^{246} = \frac{1}{2^{123}}\left(\cos\frac{246\pi}{12}+i\sin\frac{246\pi}{12}\right) = \frac{1}{2^{123}}\left(\cos(20+\frac{1}{2})\pi+i\sin(20+\frac{1}{2})\pi\right)$

$= \frac{1}{2^{123}}\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right) = \frac{i}{2^{123}}$

2. Use the division of complex numbers to calculate $\cos\frac{\pi}{12}$.

Solution. $\frac{\pi}{12} = (\frac{4}{12}-\frac{3}{12})\pi = \frac{\pi}{3}-\frac{\pi}{4}$. Now we can use the multiplication lemma: $\cos\frac{\pi}{12}+i\sin\frac{\pi}{12} =$

$= \frac{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}{\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}} = \frac{\frac{1}{2}+i\frac{\sqrt{3}}{2}}{\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}} = \frac{\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)} = \left(\frac{\sqrt{2}}{4}+\frac{\sqrt{6}}{4}\right)+\left(\frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4}\right)i$. Hence the answer is $\cos\frac{\pi}{12} = \frac{\sqrt{2}+\sqrt{6}}{4}$.

3. $\mathbf{R}^+$ is the set of positive real numbers. Prove that ($\mathbf{R}^+$,×) is a vector space over (**R,+,×)** where + and × denote ordinary addition and multiplication, and scaling by *p* is defined as raising to the power of *p*, i.e. $pv=v^p$. *You may skip showing that* ($\mathbf{R}^+$,×) *is an abelian group, but you must tell me what is the zero vector* θ.
Solution. Vectors are positive numbers, vector addition is number multiplication, so the zero vector θ=1. The result of scaling is a vector (a positive number risen to any power yields a positive number). What remains to be verified is axioms involving both vectors and scalars.
"associativity": (pq)v=p(qv). In our case it means $(pq)v=v^{pq}=v^{qp}=(v^q)^p=p(v^q)=p(qv)$, indeed,
one "distributivity": $(p+q)v=v^{p+q}=v^pv^q=pv\times qv$, and ×is vector addition,
the other "distributivity": $p(v+u)=(v\times u)^p=v^p\times u^p=pv+pu$,
scaling by 1: $1v=v^1=v$.

4. Verify if the set S={(1,0,1,1),(2,0,-1,1),(2,0,1,-1)} is linearly independent in $\mathbf{R}^4$.
Solution. Let a(1,0,1,1)+b(2,0,-1,1)+c(2,0,1,-1)=(0,0,0,0). This leads to a linear system
$\begin{cases} a+2b+c=0 \\ 0a+0b+0c=0 \\ a-b+c=0 \\ a+b-c=0 \end{cases}$. Adding equations 3 and 4 side to side we get 2*a*=0, i.e. *a*=0. Subtracting equation 3 from equation 1 we get 3*b*=0, i.e. *b*=0. Plugging *a*=*b*=0 into equation 1 we get *c*=0.
Hence the answer is YES, the set is linearly independent.