YEAR 2006f

1. F: $\mathbf{R}^3 \to \mathbf{R}^3$, F rotates every vector around the X axis by π, doubles its length and then mirrors it in the XY plane. Find the formula for F in the form F(x,y,z) = (ax+by+cz, …).

2. Solve the equation $\det\begin{bmatrix} -1-x & 2-x & -1-x & 2-x \\ 2-x & -3-x & -2-x & -2-x \\ -x & -x & -x & 1-x \\ -1-x & 2-x & 2-x & 1-x \end{bmatrix} = 0$.

   Hint. Subtract r_3 from other rows then subtract c_1 from other columns. That will leave you with only one (-*x*) in position (3,1). Then just calculate the determinant the standard way (as if *x* was a number)

3. $A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 2 & 3 & -2 & 4 \\ -1 & -1 & 1 & -1 \\ -2 & -3 & 3 & -4 \end{bmatrix}$. Find A^{-1}.

YEAR 2007f

1. F(1,1,0)=(3,4,3), F(1,0,1)=(-1,4,-1), F(0,1,1)=(0,2,-2). Find the matrix of F in the standard basis of $\mathbf{R}^3$.

2. Calculate $\det\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

   Hint. Use Laplace expansion with respect to column one.

3. Find the nullity of F(x,y,z,t)=(x+2y+z-2t,2x+3y+4z+t,3x+y+z,4y+4z-t).

4. Find A^{-1}, where $A=\begin{bmatrix} -1 & 0 & -1 & 0 \\ 1 & 1 & 2 & 0 \\ 1 & 1 & 2 & 1 \\ 1 & 2 & 4 & 1 \end{bmatrix}$.

YEAR 2007s

1. W and U are subspaces of V, $W \cap U=\{\Theta\}$, $\{w_1,\dots,w_k\}$ is a linearly independent subset of W, $\{u_1,\dots,u_m\}$ is a linearly independent subset of U. Prove that the set $\{w_1,\dots,w_k\}\cup\{u_1,\dots,u_m\}$ is linearly independent.

Hint. Suppose $a_1w_1+\dots+a_kw_k+ b_1u_1+ \dots +b_ku_m=\Theta$. This means $a_1w_1+\dots+a_kw_k=-b_1u_1- \dots -b_ku_m$. The vector on the left hand side belongs to W, the other one to U. But they are equal, so they belong to $W\cap U$. Now use the fact that $W\cap U=\{\Theta\}$.

2. Solve the equation $\det\begin{bmatrix} 1 & 1 & 1 & x \\ 1 & 1 & x & 1 \\ 1 & x & 1 & 1 \\ x & 1 & 1 & 1 \end{bmatrix} = 0$.

Hint. This is similar to 2006f-2. Row – reduce the matrix cleverly and calculate the determinant. Equate the resulting expression to zero.

3. Find all eigenvalues for $A=\begin{bmatrix} -1 & -1 & -2 & 1 \\ 4 & 3 & 4 & -2 \\ 4 & 5 & 2 & -2 \\ 6 & 5 & 4 & -2 \end{bmatrix}$.

4. $\varphi(x,y,z,t)=(t,x,y,z)$, $R=\{(1,0,1,0),(0,1,0,1),(,1,1,0),(0,1,1,1)\}$ is a basis for $\mathbf{R}^4$. Find $M_R(\varphi)$.

YEAR 2008s

1. $F(x,y,z)=(2x+y-z,x-2y+z,-x+y-2z)$. Find the matrix of F in the basis $S=\{(2,1,0),(1,3,2),(2,2,1)\}$.

2. Calculate $\det\begin{bmatrix} 1 & 2 & 3 & \cdots & n-1 & n \\ 2 & 3 & 4 & \cdots & n & 0 \\ 3 & 4 & \cdots & n & 0 & 0 \\ \vdots & \vdots & \unicode{x22F0} & \unicode{x22F0} & & \vdots \\ n-1 & n & 0 & 0 & & \vdots \\ n & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$.

   Hint. Use Laplace expansion with respect to the last row, and again, and again …

3. Solve the matrix equation AX=B, where $A=\begin{bmatrix} 0 & 3 & 3 & 1 \\ 1 & 1 & 2 & 0 \\ 1 & 2 & 2 & 1 \\ 1 & 3 & 4 & 1 \end{bmatrix}$ and $B=\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -2 \end{bmatrix}$.

Hint. This is equivalent to solving four systems of equations with each column of B playing the role of the column of constants. The solutions are columns of X. You can take a clever (but not very) shortcut.

YEAR 2008f

1. The set $\{v_1,\dots,v_k, \dots ,v_n\}$is linearly independent. Prove that $\mathrm{span}(v_1, \dots ,v_k)\cap\mathrm{span}(v_{k+1},\dots,v_n)=\{\Theta\}$.
   Hint. Do it by contradiction. Suppose there is a nonzero vector in $\mathrm{span}(v_1, \dots ,v_k)\cap\mathrm{span}(v_{k+1},\dots,v_n)$. Think!
2. Prove that if a linear operator F is 1-1 then for every linearly independent set $\{v_1, \dots ,v_k\}$ the set $\{F(v_1), \dots ,F(v_k)\}$ is linearly independent, too.
3. Find dimW where W = span{(2,3,-1,-4),(3,1,6,7),(1,1,2,3),(2,1,-1,-6)}.
   Hint. Form a matrix from this vectors and find its rank.

YEAR 2009f

2. Solve the equation $\det\begin{bmatrix} x & 1 & 1 & 1 \\ 1 & x & 1 & 1 \\ 1 & 1 & x & 1 \\ 1 & 1 & 1 & x \end{bmatrix}=0$.

YEAR 2010s

3. Calculate coordinates of (1,2,3,4) with respect to the basis {(1,1,1,0),(1,1,0,1),(1,0,1,1), 0,1,1,1)}
Hint. Just find the scalars a,b,c,d such that (1,2,3,4)=a(1,1,1,0)+b(1,1,0,1)+c(1,0,1,1)+d 0,1,1,1). If you don't know what that means I cannot help you.