Midterm 1, year 2004

1. Calculate $\left(\frac{\sqrt{2}+i\sqrt{2}}{-\sqrt{3}+i}\right)^{2004}$
2. Find all complex numbers z satisfying the equation $i\bar{z}z^3 = 8|z|$
3. Let (G,#) be a group. Consider (G^X,*) where X is any set and $(\forall x\in X)$(f*g)(x) = f(x)#g(x). Prove that (G^X,*) is a group.
4. Is the set of all odd integers a group with respect to the operation * defined as k*m=k+m-3?

Midterm 1, year 2005

1. Calculate $\left(\frac{1+\sqrt{3}+(1-\sqrt{3})i}{2+2i}\right)^{59}$
2. Find all complex numbers z satisfying the equation $(z-2+j)^4+1=0$
3. Verify if ($\mathbf{R}^+$,*) is a group, where $a*b=a^2b^2$.
4. Verify if $\mathbf{Q}(\sqrt[3]{2})$ is a field with respect to ordinary addition and multiplication. If the answer is YES, verify if it is isomorphic to $\mathbf{Q}(\sqrt{2})$.

Midterm 1, year 2006

1. Is ($\mathbf{R}$,#, $\vartriangle$) a field where, for every x,y from $\mathbf{R}$, $x\#y= x+y+1$ and $x\vartriangle y = xy+x+y$?
2. Find all complex numbers satisfying $z^5 = \bar{z}$. *Hint*: Multiply both sides by z.
3. Calculate $\left(\frac{\sqrt{2}+i\sqrt{2}}{1+i\sqrt{3}}\right)^{100}$
4. Find a basis of $W=\{(x,y,z,t)\in \mathbf{R}^4: 2x+y-3z+t=0 \wedge x-y+2z-t=0\}$.

Midterm 1, year 2007

5. 1-i is a root of $z^4-6z^3+16z^2-20z+12$. Find the remaining roots.
6. Is the set $\{x^2,2^x,\cos x\}$ linearly independent (over the field $\mathbf{R}$)?
7. Let $X=\{z\in \mathbf{C}: |z|\geq 1\}$. Is X a group with respect to multiplication of complex numbers?
8. Use complex numbers to calculate $\sin\frac{\pi}{12}$. *Hint: Use division of complex numbers.*

Midterm 1, year 2008

1. Find all complex numbers satisfying $z^3 = \bar{z}$. **Hint**: *multiply both sides by z and compare absolute values.*
2. **C** is the set of complex numbers, * is regular multiplication and x#y=(-x)+(-y). Is (**C**,#,*) a field?
3. Represent the complex number $\left(\frac{\sqrt{2}+i\sqrt{2}}{1-i\sqrt{3}}\right)^{768}$ in the standard form a+b*i*. **Hint** $\sin\frac{\pi}{4}=\frac{\sqrt{2}}{2}$, $\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$
4. Let U and W be subspaces of a vector space V over a field **F**. Show that $U\cap W$ is a subspace of V.

Midterm 1, year 2009

1. Let $z=r(\cos\alpha+i\sin\alpha)$, $w=t(\cos\beta+i\sin\beta)$. Show that $\frac{z}{w}=\frac{r}{t}(\cos(\alpha-\beta)+i\sin(\alpha-\beta))$. Calculate $\cos\frac{\pi}{12}$.
2. Find all complex numbers z satisfying the equation $i(\bar{z})^3 z = 8|z|$
3. V is a vector space over a field F. Show that for every two subspaces W and U of V, $W\cap U$ is a subspace of V. Show that there exist such V, W and U that $W\cup U$ is not a subspace of V.
4. Prove that if $\{v_1,v_2, \dots v_n\}$ is linearly independent then so is $\{v_1,v_1+v_2,v_1+v_2+v_3, \dots ,v_1+ \dots +v_n\}$.