DIMENSION

Problem 1. Determine whether the given set of vectors is linearly independent in the indicated vector space.

(a) $\{x^4+x^3+x^2+x+1, x^2+x+2, x\}$ in **R**[x] over **R,**

(b) $\{(1,0,1),(1,1,0),(0,1,1)\}$ in $\mathbf{Z}_2^3$ over $\mathbf{Z_2}$,

(c) $\{(1,0,1),(1,1,0),(0,1,1)\}$ in $\mathbf{R^3}$ over **R**,

(d) {sin x, cos x, x} in $\mathbf{R^R}$ over **R**,

(e) {$\sin^2$ x, $\cos^2$ x, sin 2x, cos 2x} in $\mathbf{R^R}$ over **R**,

(f) $\{x_1, x_1+x_2, x_1+x_2+x_3, \ldots, x_1+\ldots+x_n\}$ if $\{x_1, x_2, x_3, \ldots, x_n\}$ is linearly independent.

(g) $\{x_1+x_2, x_2+x_3, \ldots, x_{n-1}+x_n, x_n+x_1\}$ if $\{x_1, x_2, x_3, \ldots, x_n\}$ is linearly independent

(h) $\{x_1+v, x_2+v, \ldots, x_n+v\}$ if $\{x_1, x_2, x_3, \ldots, x_n\}$ is linearly independent and v is any vector,

(i) S, where S⊆T and T is linearly independent,

(j) S, where T⊆S and T is linearly independent,

(k) $\{x_1, x_2, x_3, \ldots, x_n\}$, where $x_i \in span(S_i)$, $x_i \neq \Theta$ for i=1,2, ... ,n, sets S_i are finite and pairwise disjoint and the set $S_1 \cup S_2 \cup \ldots \cup S_n$ is linearly independent.

Problem 2. Find the dimension of each of the following vector spaces:

(a) **R** over **Q**,

(b) **C** over **R**,

(c) All polynomials with real coefficients, of degree ≤7, with a root at 1, over **R**,

(d) $\mathbf{F}^n$ over **F**, where **F** is any field,

(e) All polynomials with real coefficients, of degree ≤8, divisible by x^2+1, over **R.**

(f) *Span*({$\sin^2$ x, $\cos^2$ x, sin 2x, cos 2x}) over **R**,

(g) *Span*($\{x_1, x_2, x_3, \ldots, x_{n-1}\}$), where $\{x_1, x_2, x_3, \ldots, x_n\}$ is a basis for V,

(h) *Span*($\{x_n, x_{n-1}, x_{n-2}, \ldots, x_1\}$), where $\{x_1, x_2, x_3, \ldots, x_n\}$ is a basis for V,

Problem 3. Let $U=\{(a,b,c,d)\in \mathbf{R^4} | b-2c+d=0\}$ and $W=\{(a,b,c,d) \in \mathbf{R^4} | a=d \ \& \ b=2c\}$.Find the dimension of U,W and of U∩W.

Problem 4. Prove that for every subset S of a vector space V:

(a) S⊆*span*(S),

(b) S⊆T ⇒ *span*(S) ⊆ *span*(T),

(c) *span*(S) = *span*(S∪{w}), for every vector w∈*span*(S),

(d) *span*(*span*(S)) = *span*(S),

Problem 5. Let **V** be a vector space over a field **F**. Let SUB(**V**) denote the set of all subspaces of **V**. For every **U,W**∈SUB(**V)** and for every scalar *p*∈**F,** let **U+W** = {u+w : u∈**U** & w∈**W**} and *p***W** = {*p*w : w∈**W**}.

(a) Prove that “+” is an operation in the set SUB(**V**)?

(b) Is (SUB(**V**),+) a group? Is it commutative?

(c) Prove that *p***W** is a subspace of **V**.

(d) Prove that every linear combination of subspaces of **V** is a subspace of **V**.

(e) Let $\mathbf{W_1}, \mathbf{W_2}, \ldots, \mathbf{W_k}$ be subspaces of **V**. Prove that $dim(span(\mathbf{W_1}, \mathbf{W_2}, \ldots, \mathbf{W_k})) \leq dim(\mathbf{W_1}) + dim(\mathbf{W_2}) + \ldots + dim(\mathbf{W_k})$.