FIELDS AND COMPLEX NUMBERS

Problem 1. Show that multiplication of complex numbers is associative and distributive.

Problem 2. Determine which of the following algebras are fields

(a) $(2^X, \cup, \cap)$
(b) $(2^X, \cap, \cup)$
(c) $(2^X, \cap, \div)$
(d) $(2^X, \cup, \div)$
(e) $(2^X, \div, \cup)$
(f) $(2^X, \div, \cap)$
(g) $(\mathbf{R}^{\#}, \times, +)$
(h) $(R^R, +, \times)$

Problem 3. Are the fields **C** and **R** isomorphic?

Problem 4. Verify that both $\mathbf{Q}(\sqrt{2})$ and $\mathbf{Q}(\sqrt{3})$ are fields. Are they isomorphic?

Problem 5. Show that $(\mathbf{Z}_n, \oplus, \otimes)$ is a field iff *n* is a prime.

Problem 6. Calculate

(a) $\left|\frac{1+4i}{4-i}\right|$

(b) $\left|\frac{z}{\bar{z}}\right|$

(c) $\left|\frac{(3+4i)^4}{(3-4i)^3}\right|$

Problem 7. Find all roots of unity of order 6.

Problem 8. Show that for every *n* all roots of unity of order *n* form a group under multiplication. What well-known group is it isomorphic to?

Problem 9. Express the following complex numbers in the standard form a+b*i*.

(a) $\frac{1+i}{1-i}$

(b) $\frac{2+i}{1+2i}$

(c) $\sqrt[3]{(1+i)^6}$

Problem 10. Find polar forms for the following complex numbers

(a) $3+(3\sqrt{3})i$

(b)-3 (negative 3)

(c) $2+2i$

(d) $1-i\sqrt{3}$

Problem 11. Let $f(x) \in \mathbf{R}[x]$. Show that *z* is a root of *f* iff $\bar{z}$ is a root of *f*.

Problem 12. Factor each of the following polynomials into a product of factors of degree at most 2.

(a) x^6+1

(b) x^4+1

(c) x^6+x^3+1