TUTORIAL 10. DETERMINANT. INVERSE MATRIX.

1. Calculate the following determinants:

(a) $\begin{vmatrix} 1 & 2 & -2 & 1 \\ -3 & 1 & 3 & 2 \\ -1 & 2 & 4 & 2 \\ 5 & -1 & 2 & 2 \end{vmatrix}$ (b) $\begin{vmatrix} 1 & 2 & 1 & 3 \\ 2 & 1 & 2 & 1 \\ 3 & 2 & 3 & 2 \\ 4 & 3 & 4 & 3 \end{vmatrix}$ (c) det(A), where A(i,j)=i-j, n=5

2. Show that $\det(A)=\det(A^T)$.
3. Show that det(A)=0 if and only if one of the rows is a linear combination of the others.
4. Show that for every 2×2 matrices A and B, det(AB)=det(A)det(B).
5. Show that $(AB)^{-1}=B^{-1}A^{-1}$.
6. Calculate A^{-1}

(a) $A=\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ (b) $A=\begin{bmatrix} -1 & 0 & -1 & 0 \\ 1 & 1 & 2 & 0 \\ 1 & 1 & 2 & 1 \\ 1 & 2 & 4 & 1 \end{bmatrix}$ (c) $A=\begin{bmatrix} -1 & 2 & -1 & 2 \\ 2 & -3 & -2 & -2 \\ 0 & 1 & - & 1 \\ -1 & 2 & 2 & 1 \end{bmatrix}$

(d) $A=\begin{bmatrix} 2 & 3 & 2 & 0 & 1 \\ 0 & 1 & 1 & 1 & -1 \\ 1 & 2 & 1 & 0 & 0 \\ 0 & 3 & 1 & 1 & -2 \\ 2 & 2 & 1 & 0 & 1 \end{bmatrix}$ (e) $A=\begin{bmatrix} 7 & -1 & -7 & 1 \\ 0 & -1 & 1 & -2 \\ -5 & 0 & 6 & -2 \\ 1 & 1 & -2 & 2 \end{bmatrix}$ (f) $A=\begin{bmatrix} 0 & 1 & 1 & -1 \\ 2 & 2 & 1 & -1 \\ 5 & 6 & 1 & -2 \\ -4 & -5 & -1 & 2 \end{bmatrix}$

7. Prove that for every n×n matrix A over a field **F**, $det(A-\lambda I) \in \mathbf{F}_n[\lambda]$.