TUTORIAL 11. LINEAR MAPPINGS. MATRICES OF LINEAR MAPPINGS.

1. Find Ker F, Im F, rank F and nullity F for the following linear mappings F:U→V:
   - (a) U=V=$\mathbf{R^3}$, F(x,y,z)=(x+y+z,x+y,x)
   - (b) U=V=$\mathbf{R^3}$, F(x,y,z)=(x+y,x+y,x+2y-z)
   - (c) U=V=$\mathbf{R_n}$ [x], F(w(x))=w(x)+w'(x)
   - (d) U=V=span{1,sin x,cos x,exp x}, F(g)(x)=g'(x)
   - (e) U=$\mathbf{R}^3$,V=$\mathbf{R}^2$, F(x,y,z)=(x-y+z,x+y-z)
2. Find the matrices of the linear mappings from Problem 3 a,b,c,d,e in the following bases:
   (a) the usual basis of unit vectors
   (b) {(1,1,1),(1,1,0),(1,0,0)} and {(1,0,0),(1,1,0),(1,1,1)},
   (c) {1,x,x^2,...,x^n} and {1,1+x,1+x+x^2 ,...,1+x+x^2+...+x^n}
   (d) {1,sin x,cos x,exp x} (show that this is a basis!)
   (e) {(1,1,1),(1,0,0),(0,0,1)} and {(1,2),(3,3)}
3. Let T:C→C, T(z)= $\bar{z}$.
   (a) Show that T is NOT linear if we consider C a vector space over itself
   (b) Show that T is linear if we consider C a vector space over R
4. Find a formula and the matrix in the standard basis for the linear operator F such that:
   (a) F(1,1,1)=(1,0,1), F(1,1,0)=F(0,1,1)=(1,1,1)
   (b) F(1,2,1)=(1,0,0), F(1,0,1)=(0,1,0), F(1,0,0)=(1,0,0)
   (c) F(1,0,0,0)=F(0,1,0,0)=F(0,0,1,0)=F(0,0,0,1)=(1,1,1,1)