TUTORIAL 12. MATRICES OF LINEAR MAPPINGS. CHANGE OF BASIS

1. Assuming that A is the matrix of a linear operator F in S find the matrix B of F in R:

(a) $A=\begin{bmatrix}1 & 1\\ 1 & 2\end{bmatrix}$, S={(1,1),(1,2)}, R={(1,0),(0,1)}

(b) $A=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$, S={(1,1,1),(0,1,2),(1,0,1)}, R={(1,0,0),(0,1,0),(0,0,1)}

(c) $A=\begin{bmatrix}1 & 2 & 2\\ 1 & 1 & 1\\ 2 & 0 & 0\end{bmatrix}$, S={(1,1,1),(0,1,2),(1,0,1)}, R={(1,0,0),(0,1,0),(0,0,1)}

(d) $A=\begin{bmatrix}1 & 2 & 3\\ 2 & 1 & 0\\ 3 & 0 & 1\end{bmatrix}$, S={(0,0,1),(0,1,0),(1,0,0)}, R={(1,1,0),(1,1,1),(0,1,1)}

(e) $A=\begin{bmatrix}1 & 1 & 1 & 1\\ 0 & 1 & 1 & 1\\ 0 & 0 & 1 & 1\\ 0 & 0 & 0 & 1\end{bmatrix}$, S={(1,0,1,0),(0,0,1,1),(1,1,0,0),(1,0,1,1)},
R={(1,1,1,0),(1,1,0,1),(1,0,1,1),(0,1,1,1)}

2. For each two matrices A and B from problem 1 find the change-of-basis matrix P such that $A=P^{-1}BP$. Verify your solution by matrix multiplication.
3. For each operator from problem 1 determine if there exists a basis T such that the matrix of the operator in T is diagonal.
4. Find all eigenvalues and a basis of each eigenspace for the operator $F:R^3 \to R^3$, $F(x,y,z)=(2x+y,y-z,2y+4z)$
5. For each of the following matrices find all eigenvectors and a basis for each eigenspace.

$(a)\ A=\begin{bmatrix}3 & 1 & 1\\ 2 & 4 & 2\\ 1 & 3 & 3\end{bmatrix}\quad (b)\ B=\begin{bmatrix}1 & 2 & 2\\ 1 & 2 & -1\\ -1 & 1 & 4\end{bmatrix}\quad (c)\ C=\begin{bmatrix}1 & 1 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$