TUTORIAL 13. JORDAN BLOCK MATRICES

1. Assuming that each matrix represents a linear operator T find a basis R such that $M_R(T)$ is a diagonal matrix D, resp. In each case find a change of basis matrix P (i.e. such that $D=P^{-1}AP$).

$$\mathbf{A} = \begin{bmatrix} -1 & -2 & -1 & 1 \\ 0 & 1 & 1 & -1 \\ 8 & 6 & 3 & 0 \\ -4 & -6 & 1 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 2 & -4 & -1 & 1 \\ 2 & 1 & -2 & -2 \\ 0 & -2 & 1 & 1 \\ 2 & -2 & -2 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 2 & 1 & 1 & -1 \\ -2 & -1 & -2 & 2 \\ -2 & -2 & -1 & 2 \\ -4 & -4 & -4 & 5 \end{bmatrix}$$

$$D = \begin{bmatrix} 2 & -2 & 1 \\ 1 & 5 & -1 \\ 2 & 4 & 1 \end{bmatrix} \quad E = \begin{bmatrix} 2 & -2 & 1 \\ 1 & 5 & -1 \\ 2 & 4 & 1 \end{bmatrix} \quad F = \begin{bmatrix} 1 & 0 & 1 \\ -2 & 2 & 2 \\ 2 & -1 & 1 \end{bmatrix}$$

2. Assuming that each matrix represents a linear operator T find a basis R such that $M_R(T)$ is a Jordan block matrix J. Also, in each case find the change of basis matrix (i.e. such matrix P that $J=P^{-1}AP$).

$$A=\begin{bmatrix} 0 & -2 & -1 & 1 \\ 2 & 4 & 1 & -1 \\ 2 & 2 & 3 & -1 \\ 2 & 2 & 1 & 1 \end{bmatrix}, \quad B=\begin{bmatrix} 0 & -1 & 0 & 1 \\ 1 & 1 & 2 & -1 \\ 0 & -1 & 3 & 0 \\ -1 & -2 & 3 & 1 \end{bmatrix}, \quad C=\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 1 & 3 \\ 0 & 0 & 2 & 5 \\ 0 & 0 & 0 & 2 \end{bmatrix}, \quad D=\begin{bmatrix} -3 & -4 & 1 & 7 & -8 \\ -1 & -2 & 0 & 3 & -3 \\ -1 & -3 & 0 & 4 & -3 \\ -1 & -3 & -1 & 5 & -3 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$E=\begin{bmatrix} 1 & -1 & -1 & 1 \\ 1 & 3 & 1 & -1 \\ 4 & 5 & 3 & -2 \\ 3 & 4 & 1 & 0 \end{bmatrix}, \quad F=\begin{bmatrix} 7 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ 3 & 3 & -5 & 0 \\ -1 & -2 & 3 & 3 \end{bmatrix}$$