

TUTORIAL 2. COMPLEX NUMBERS 2

2.1. Calculate:

(a) $\sqrt[3]{(1+i)^6}$

(b) $\left(\frac{1-i}{1+i}\right)^{222}$

(c) $\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^{124}$

(d) $\left(\frac{2+2i}{\sqrt{3}-i}\right)^8$

(e) $\sqrt[6]{1}$

(f) $\sqrt[4]{-1}$

(g) $\sqrt[4]{(3-5i)^4}$

(i) $\sqrt[6]{-i}$

2.2. Use de Moivre Law to

(a) express $\sin 5\alpha$ in terms of $\sin \alpha$ and $\cos \alpha$,

(b) express $\cos 4\alpha$ in terms of $\sin \alpha$ and $\cos \alpha$.

2.3. Prove that $\frac{\cos \alpha + i \sin \alpha}{\cos \beta + i \sin \beta} = \cos(\alpha - \beta) + i \sin(\alpha - \beta)$.

2.4. Find the polar form of

(a) $2 + \sqrt{2} + i\sqrt{2}$

(b) $2 + i + i\sqrt{3}$

2.5. Calculate

(a) $\cos \frac{\pi}{12}$

(b) $\cos 105^\circ$

(c) $\sin 75^\circ$

(d) $\sin(-15^\circ)$

2.6. Solve equations

(a) $x^2 + x + 1 = 0$

(b) $2x^2 + ix + 3 = 0$