TUTORIAL 4. FIELDS AND GROUPS

4.1. Prove

(a) $(\forall n,k)\ (n\,mod\,k)\,mod\,k = n\,mod\,k$

(b) $(\forall n,k,p)\ (n+p)\,mod\,k = (n\,mod\,k + p\,mod\,k)\,mod\,k$

(c) $(\forall n,k,p)\ (np)\,mod\,k = ((n\,mod\,k)(p\,mod\,k))\,mod\,k$

4.2. Prove

(a) multiplication *mod n* is associative, i.e. $(a\otimes b)\otimes c=a\otimes(b\otimes c)$.

(b) addition *mod n* is associative.

(c) multiplication *mod n* is distributive with respect to addition *mod n*.

4.3. Calculate, solve equations

(a) $17\mod 4$ (b) $4\mod 17$ (c) $(-2)\mod 5$

(d) $(2x=1)\mod 7$ (e) $(2x=1)\mod 6$ (f) $(5x=1)\mod 9$

(g) $(x^2=3)\mod 11$ (h) $(2x+3=0)\mod 5$ (i) $(x+k=0)\mod n$

4.4. Determine which of the following algebras are fields. Do this in two steps : first verify if the set is a group with respect to the first operation, and next if the set without the identity element of the first operation is a group with respect to the second operation.

(a) $(2^X,\cup,\cap)$ (b) $(R^R,+,\times)$

(c) $(2^X,\cap,\cup)$ (d) $(\mathbf{R}^\#,\times,+)$

(e) $(2^X,\cap,\div)$ (f) $(2^X,\div,\cap)$

(g) $(2^X,\cup,\div)$ (h) $(2^X,\div,\cup)$

4.5. Show that $(\mathbf{Z}_n-\{0\},\otimes)$ is a group iff *n* is a prime.

4.6. Show that for every *n* all complex roots of 1 of order *n* form a group under multiplication.

4.7. Show that all complex roots of 1 of all integer orders form a group under multiplication.

4.8. Verify if $(\mathbf{R}^+,\#)$ is a group, where $a\#b=a^2b^2$.

4.9. Show that if $(\mathbf{F},\#,\&)$ is a field then for every positive integer k $(\mathbf{F}^k,\%)$ is a group, where $(a_1,a_2,\dots,a_k)\%(b_1,b_2,\dots,b_k) = (a_1\#b_1, a_2\#b_2,\dots,a_k\#b_k)$.