TUTORIAL 5. VECTOR SPACES

5.1. Determine whether or not the following sets are vector spaces over indicated fields:

a. Z over Z_2, with ordinary addition, and scaling defined as 0r=0, 1r=r for every $r \in Z$,

b. Z_4 over Z_2 with natural operations,

c. C over R,

d. R over C,

e. $Q_n[x]$ (all polynomials from Q[x] with degree $\leq n$) over R,

f. $Q_n[x]$ over Q,

g. All polynomials from R[x] with exactly two different roots, over R,

h. $R_n[x]$ (all polynomials from R[x] with degree $\leq n$) over R,

i. All subsets of a set X, over Z_2, with the symmetric difference as vector addition, and with scaling defined as $0A=\varnothing$ and 1A=A for every $A \subseteq X$,

j. $(\mathbf{R}^+, \times)$ over $(\mathbf{R}, +, \times)$ where + and $\times$ denote ordinary addition and multiplication, and scaling defined as $p \cdot v = v^p$.

5.2. Prove that in a vector space $p \cdot (v-u) = p \cdot v - p \cdot u$.

5.3. The intersection of any collection of subspaces of some linear space V is a subspace of V. Show that this is not necessarily true for the union of subspaces.

5.4. The real plane, R^2, is a vector space over R. Describe, in geometrical terms, all subspaces of R^2.

5.5. Prove that

a. $S \subseteq \mathit{span}(S)$

b. $S \subseteq T \Rightarrow \mathit{span}(S) \subseteq \mathit{span}(T)$

c. $\mathit{span}(\mathit{span}(S)) = \mathit{span}(S)$

d. $v \in \mathit{span}(S) \Leftrightarrow \mathit{span}(S) = \mathit{span}(S \cup \{v\})$