TUTORIAL 6. LINEAR INDEPENDENT SETS. SPANNING SETS.

6.1. Decide whether the given set of vectors is linearly independent in the indicated vector space.

a. $\{ x^4 + x^3 + x^2 + x + 1, x^2 + x + 2, x \}$ in **R**[x] over **R,**
b. {(1,0,1),(1,1,0),(0,1,1)} in $\mathbf{Z}_2^3$ over $\mathbf{Z}_2$,
c. {(1,0,1),(1,1,0),(0,1,1)} in $\mathbf{R}^3$ over **R**,
d. {sin x, cos x, x} in $\mathbf{R}^{\mathbf{R}}$ over **R**,
e. {$\sin^2$ x, $\cos^2$ x, sin 2x, cos 2x} in $\mathbf{R}^{\mathbf{R}}$ over **R**,
f. $\{ x_1, x_1 + x_2, x_1 + x_2 + x_3, \ldots, x_1 + \ldots + x_n \}$ if $\{ x_1, x_2, x_3, \ldots, x_n \}$ is linearly independent, in any vector space V,
g. $\{ x_1 + x_2, x_2 + x_3, \ldots, x_{n-1} + x_n, x_n + x_1 \}$ if $\{ x_1, x_2, x_3, \ldots, x_n \}$ is linearly independent, in any vector space V,
h. $\{x_1 + v, x_2 + v, \ldots, x_n + v\}$ if $\{ x_1, x_2, x_3, \ldots, x_n \}$ is linearly independent and v is any vector,
i. S, where S⊆T and T is linearly independent,
j. S, where T⊆S and T is linearly independent,
k. $\{ x_1, x_2, x_3, \ldots, x_n \}$, where $x_i \in span(S_i)$, $x_i \neq \Theta$ for i=1,2, ... ,n, sets S_i are finite and pairwise disjoint and the set $S_1 \cup S_2 \cup \ldots \cup S_n$ is linearly independent.

6.2. Find the dimension of each of the following vector spaces:

a. **R** over **Q**,
b. **C** over **R**,
c. All polynomials with real coefficients, of degree ≤7, with a root at 1, over **R**,
d. $\mathbf{F}^n$ over **F**, where **F** is any field,
e. All polynomials with real coefficients, of degree ≤8, divisible by x^2+1, over **R.**
f. *Span*({$\sin^2$ x, $\cos^2$ x, sin 2x, cos 2x}) over **R**,
g. *Span*($\{x_1, x_2, x_3, \ldots, x_{n-1}\}$), where $\{ x_1, x_2, x_3, \ldots, x_n \}$ is a basis for V,
h. *Span*($\{x_n, x_{n-1}, x_{n-2}, \ldots, x_1\}$), where $\{ x_1, x_2, x_3, \ldots, x_n \}$ is a basis for V,
i. $\{(x,y,z,t) \in \mathbf{R}^4 \mid x+y+z+t=0\}$,
j. $\{(x,y,z,t) \in \mathbf{R}^4 \mid 2x+y=z-t\}$,
k. *span*{(1,2,1,0),(1,3,1,1),(0,1,2,-1),(-3,1,2,2)} over **R**,
l. $2^{\{a,b,c\}}$ over $\mathbf{Z}_2$.