Tutorial 7. Bases and Dimension.

7.1. Find the dimension of each of the following vector spaces:

a. **R** over **Q**,
b. **C** over **R**,
c. All polynomials with real coefficients, of degree ≤7, with a root at 1, over **R**,
d. $\mathbf{F}^n$ over **F**, where **F** is any field,
e. All polynomials with real coefficients, of degree ≤8, divisible by x^2+1, over **R.**
f. *Span*({$\sin^2$ x, $\cos^2$ x, sin 2x, cos 2x}) over **R**,
g. *Span*($\{x_1, x_2, x_3, ..., x_{n-1}\}$), where $\{x_1, x_2, x_3, \ldots, x_n\}$ is a basis for V,
h. *Span*($\{x_n, x_{n-1}, x_{n-2}, ..., x_1\}$), where $\{x_1, x_2, x_3, \ldots, x_n\}$ is a basis for V,
i. $\{(x,y,z,t)\in\mathbf{R}^4 \mid x+y+z+t=0\}$,
j. $\{(x,y,z,t)\in\mathbf{R}^4 \mid 2x+y=z-t\}$,
k. *span*{(1,2,1,0),(1,3,1,1),(0,1,2,-1),(-3,1,2,2)} over **R**,
l. $2^{\{a,b,c\}}$ over $\mathbf{Z}_2$.

7.2. Calculate coordinates of v with respect to the basis S.

a. v=(1,2,3,4), S={(1,1,1,1), (1,1,1,0), (1,1,0,1), (1,0,1,1)},
b. v={a,b}, S={{a},{b},{c}},
c. v=(0,1,1,1), S={(1,1,1,1), (1,1,1,0), (1,1,0,1), (1,0,1,1)},
d. $v=v_1$, S=$\{v_1,v_2, \ldots ,v_n\}$,
e. $v=v_1+v_2+ \ldots +v_n$, S=$\{v_1,v_2, \ldots ,v_n\}$.

7.3. Prove that if $\{v_1,v_2, \ldots ,v_n\}$ is a basis for a vector space V then so is $\{v_1,v_1+v_2, \ldots ,v_1+v_2+ \ldots +v_n\}$.