TUTORIAL 8. MATRICES

1. Prove that matrix multiplication is associative.

2. Prove that matrix multiplication is distributive with respect to matrix addition.

3. A square (i.e. n×n) matrix A is said to be *symmetric* iff $A=A^T$. Is it true that multiplication of symmetric matrices is commutative?

4. Show that $(AB)^T=B^TA^T$.

5. Reduce the matrices to their row echelon and row canonical forms and find their ranks

$$A=\begin{bmatrix}2&1&3&-2&1\\1&1&2&3&-3\\2&2&2&2&2\\5&7&-3&-6&4\end{bmatrix} \quad B=\begin{bmatrix}1&2&3\\4&5&6\\7&8&9\\10&11&12\end{bmatrix} \quad C=\begin{bmatrix}5&1&2&6&1&0\\4&0&1&2&1&1\\3&5&2&3&2&1\\2&4&1&-1&2&2\end{bmatrix}$$

$$D=\begin{bmatrix}1&3&-2&5&4\\1&4&1&3&5\\1&4&2&4&3\\2&7&-3&6&13\end{bmatrix} \quad E=\begin{bmatrix}1&4&5&-1\\1&5&8&-2\\2&5&1&2\end{bmatrix} \quad F=\begin{bmatrix}2&1\\3&-7\\-6&1\\5&-8\end{bmatrix}$$

6. Show that for every two n×k matrices A and B, $\text{rank}(A+B)\le\text{rank}(A)+\text{rank}(B)$.

7. Calculate the matrix AB.

a. $A=\begin{bmatrix}1&0\\2&3\\1&2\\-1&1\end{bmatrix}$, $B=\begin{bmatrix}3&-2&2\\1&5&3\end{bmatrix}$

b. $A=\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$, $B=\begin{bmatrix}1&4&7\\2&5&8\\3&6&9\end{bmatrix}$

c. $A=\begin{bmatrix}1&4&7\\2&5&8\\3&6&9\end{bmatrix}$, $B=\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$

d. $A=\begin{bmatrix}1&1&0&0\\3&2&3&1\\1&0&1&0\end{bmatrix}$, $B=\begin{bmatrix}-2&1\\1&0\\2&0\\-1&1\end{bmatrix}$

8. Use matrices to determine the dimension of W:

a. W=span{(1,2,3),(1,-1,2),(-1,4,-1)}
b. W=span{(3,1,2,1),(1,-1,1,2),(2,3,0,-3),(2,-1,1,2)}
c. W=span{(6,3,-5,3),(2,2,-2,1),(2,1,-1,1),(-8,-5,8,-4)}
d. W=span{(1,1,-1,0),(-2,-3,1,1),(-1,-3,2,1),(-4,-7,1,3)}
e. W=span{(-2,0,-2,0),(2,-3,2,2),(3,-6,3,4),(4,-9,4,6)}