TUTORIAL 9. SYSTEMS OF LINEAR EQUATIONS

1. Find a basis and the dimension to the solution space of the systems of linear equations:

$$(a)\begin{cases} x+2y-2z+2s-t=0 \\ x+2y-z+3s-2t=0 \\ 2x+4y-7z+s+t=0 \end{cases} \qquad (b)\begin{cases} x+2y-z+3s-4t=0 \\ 2x+4y-2z-s+5t=0 \\ 2x+4y-2z+4s-2t=0 \end{cases}$$

2. Let A be a square n×n matrix. Show that if AX=Θ has only the zero solution, then, for every vector B, AX=B has exactly one solution.

3. Discuss solvability of the following systems of equations in terms of parameters a and b.

$$(a)\begin{cases} ax+by+az+t=1 \\ ax+y+az+t=1 \\ x+y+az+at=1 \end{cases} \quad (b)\begin{cases} x+2y+3z+4t=a \\ 2x+3y+4z+5t=b \\ 3x+4y+5z+6t=a \\ 4x+5y+6z+7t=b \end{cases} \quad (c)\begin{cases} x+ay+az=1 \\ ax+ay+z=1 \\ ax+y+az=1 \\ ax+ay+az=1 \end{cases}$$

4. Find general solutions to the following systems of equations:

$$(a)\begin{cases} 2x+2y+3z+4t=2 \\ 2x+3y+4z+6t=0 \\ 3x+4y+5z+6t=1 \\ x+y+z+t=1 \end{cases} \quad (b)\begin{cases} x+y+z-t+u=1 \\ x-y+z+t+u=1 \\ -x+y-z+t-u=0 \end{cases} \quad (c)\begin{cases} x+y+z+t+u=0 \\ x-2y+3z-4t+5u=0 \end{cases}$$