EDDE. PROBLEM SET 8 (old version)

1. Directly applying the definition, compute the Laplace transforms of the functions $t \cdot e^{at}, t \cdot \cos at, t \cdot \sin at$. Feel free to use any formula you can remember (eg. integration by parts, Euler's identity), but not the Laplace transformation tables.

2. Prove that
$$L\left[t \cdot f(t)\right](s) = -\frac{dL\left[f(t)\right](s)}{ds}.$$
Using that identity compute again the Laplace transforms of the functions $t \cdot e^{at}, t \cdot \cos at, t \cdot \sin at$.

3. Find the inverse Laplace transforms of the functions
$$\frac{s+1}{s^2+2s}, \frac{2s^2+s+1}{s^3-s}, \frac{2s^2+s+1}{s^5-s^3}, \frac{4s^3-8s}{s^4-1}, \frac{4s^3-8s}{s^4-2s^2+1}, \frac{4s^3-8s}{s^4+2s^2+1}.$$
Remark: In many cases one can apply two or three different methods.

4. Let a be an arbitrary real number. Using the Laplace transformation solve the differential equation $x'(t)+ax(t) = e^t+a$ with the condition $x(0) = 1$.

   Remark: In some sense there are two different solutions: one for $a = -1$ and one for $a \neq -1$.