$$y' + \frac{4}{x} y = x^3 y^2 \qquad \Big| : y^2$$

$$\frac{y'}{y^2} + \frac{4}{x}\cdot\frac{1}{y} = x^3 \qquad w = \frac{1}{y} \ ; \ w' = -\frac{y'}{y^2}$$

$$(*) \quad -w' + \frac{4}{x} w = x^3 \qquad y = \frac{1}{w}$$

$$\text{HE: } \underbrace{-w' + \frac{4}{x} w = 0}$$

$$\frac{dw}{dx} = w' = \frac{4w}{x} \Rightarrow \int \frac{dw}{w} = 4\int \frac{dx}{x}$$

$$\ln|w| = 4 \ln|x| + \ln C \Rightarrow \underbrace{w = \pm C x^4}$$

$$w(x) = C(x)\, x^4 \quad (\text{VARIATION OF C})$$

$$w'(x) = C'(x)\, x^4 + C(x)\cdot 4x^3$$

$$-C'(x)x^4 - \cancel{4C(x)x^3} + \cancel{\frac{4}{x} C(x)x^4} = x^3$$

$$C'(x) = -\frac{x^3}{x^4} = -\frac{1}{x} \Rightarrow C(x) = -\ln|x| + C$$

$$w(x) = C(x)\cdot x^4 = -x^4 \ln|x| + C x^4$$

$$y(x) = \frac{1}{-x^4 \ln|x| + C x^4}$$

$n = \frac{1}{2}$

$$\begin{cases} y' + \frac{y}{x} = \sqrt{y} \quad \Big| : \sqrt{y} \\ y(1) = 0 \end{cases}$$

$$\underbrace{\frac{y'}{\sqrt{y}} + \frac{1}{x}\sqrt{y} = 1}$$

$u(x) = \sqrt{y(x)} \Rightarrow y(x) = [u(x)]^2$

$\Rightarrow u'(x) = \frac{y'(x)}{2\sqrt{y(x)}}$

$$\underline{2u' + \frac{1}{x}u = 1}$$

HE: $2u' + \frac{1}{x}u = 0$

$\frac{du}{dx} = u' = -\frac{u}{2x}$

$\int \frac{du}{u} = -\frac{1}{2}\int \frac{dx}{x}$

$\ln|u| = -\frac{1}{2}\ln|x| + \ln C$

$u = \pm C \cdot x^{-\frac{1}{2}}$

$$\begin{cases} u(x) = C(x) \cdot x^{-\frac{1}{2}} \Rightarrow \\ u'(x) = C'(x) \cdot x^{-\frac{1}{2}} - \frac{1}{2} C(x) x^{-\frac{3}{2}} \end{cases}$$

$2C'(x)x^{-\frac{1}{2}} - \cancel{C(x)x^{-3/2}} + \cancel{C(x)x^{-3/2}} = 1$

$C'(x) = \frac{1}{2}x^{1/2} \Rightarrow C(x) = \frac{1}{2} \cdot \frac{x^{3/2}}{3/2} + C = \frac{x^{3/2}}{3} + C$

$u(x) = C(x) \cdot x^{-1/2} = \frac{x}{3} + C x^{-\frac{1}{2}}$

$\frac{1}{9}x^2 - \frac{2}{9}\sqrt{x} + \frac{1}{9x}$

$\shortparallel$

$y(x) = [u(x)]^2 = \left(\frac{x}{3} + C x^{-\frac{1}{2}}\right)^2$

$0 = y(1) = \left(\frac{1}{3} + C\right)^2 \Rightarrow C = -\frac{1}{3}$

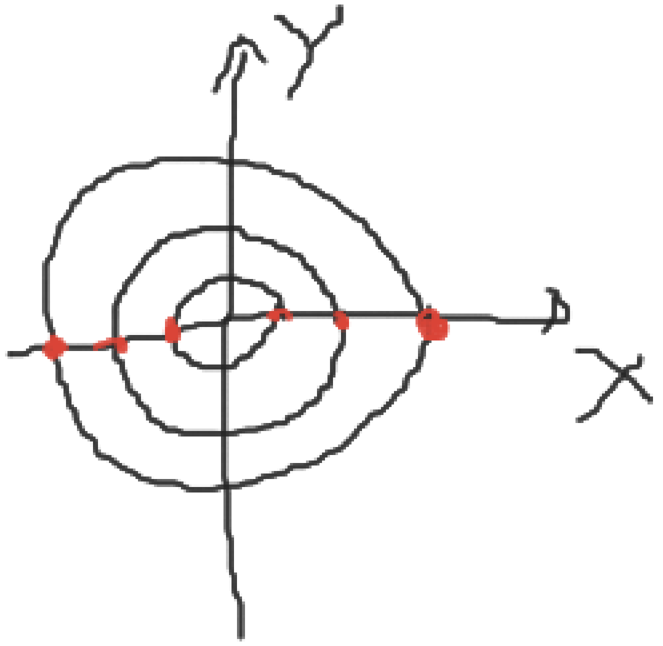
$y(x) = \left(\frac{x - x^{-\frac{1}{2}}}{3}\right)^2$

EXACT DIFFERENTIAL EQUATION

Ex. $\boxed{x^2 + y^2 = C}$

$y = \pm\sqrt{C - x^2}$

$y' = \pm \frac{1}{2\sqrt{C-x^2}} \cdot (-2x) = \mp \frac{x}{\sqrt{C-x^2}} = -\frac{x}{y}$



$\frac{dy}{dx} = \boxed{y' = -\frac{x}{y}} \Rightarrow \int y\,dy = -\int x\,dx$

$[y^2(x)]' = 2y(x)\cdot y'(x)$

$\frac{y^2}{2} = -\frac{x^2}{2} + \frac{C}{2}$

$y\,dy = -x\,dx$

$\boxed{x\cdot dx + y\cdot dy = 0} \rightarrow \boxed{x^2 + y^2 = C}$

$\frac{d}{dx}$

$\frac{d(x^2+y^2)}{dx} = \frac{dC}{dx} = 0$

$2x + 2y\frac{dy}{dx}$

$2x + 2yy' = 0$

$\frac{dy}{dx} = y' = -\frac{x}{y}$

$$\boxed{F(x,y)=C} \;=\; F(x,y(x)) \quad \Big|\Rightarrow \quad \frac{d\,F(x,y(x))}{dx} = 0$$

$$\overset{\text{CHAIN RULE}}{=} \frac{\partial F}{\partial x} + \frac{\partial F}{\partial y}\cdot y'(x)$$

$$\begin{tikzcd} & x \arrow[r] & F \\ x \arrow[r] & y \arrow[ur] & \end{tikzcd}$$

IMPLICIT SOLUTION

$\not\Downarrow$

EXPLICIT

$y(x) = \ldots\ldots$

$$\frac{\partial F}{\partial x} + \frac{\partial F}{\partial y}\cdot\frac{dy}{dx} = 0 \quad \Big|\cdot dx$$

$$\boxed{\frac{\partial F}{\partial x}\,dx + \frac{\partial F}{\partial y}\cdot dy = 0} \rightarrow \text{EXACT DIFF. EQUATION}$$

(EXACT) DIFFERENTIAL OF F

$$\underbrace{(2x-y)}_{\frac{\partial F}{\partial x}}dx + \underbrace{(2y-x)}_{\frac{\partial F}{\partial y}}dy = 0$$

$$y' = \frac{2x-y}{x-2y}$$

$$\parallel$$

$$\frac{dy}{dx}$$

$$\boxed{F = ?}$$

$$\begin{cases} \dfrac{\partial F}{\partial x} = 2x - y \implies F(x,y) = \int (2x-y)\,dx = x^2 - xy + C(y) \\ \dfrac{\partial F}{\partial y} = \underbrace{2y - x} \end{cases}$$

$$\Downarrow$$

$$\frac{\partial F}{\partial y} = \underbrace{0 - x + C'(y)}$$

$$2y - x = -x + C'(y) \implies C'(y) = 2y$$

$$\Downarrow$$

$$C(y) = y^2 + C$$

FINALLY $F(x,y) = x^2 - xy + y^2 + C$

ANSWER: $\boxed{x^2 - xy + y^2 = C}$

$$y = \frac{x \pm \sqrt{4C - 3x^2}}{2}$$

Ex $\underbrace{(x^2 - y)}_{= \frac{\partial F}{\partial x}}\,dx + \underbrace{(y^2 - 2x)}_{= \frac{\partial F}{\partial y}}\,dy = 0$

EXACT?

$$\begin{cases} \dfrac{\partial F}{\partial x} = x^2 - y \\[2ex] \dfrac{\partial F}{\partial y} = y^2 - 2x \end{cases}$$

$\Rightarrow F(x,y) = \int (x^2 - y)\,dx = \frac{x^3}{3} - yx + C(y)$

$$\underbrace{\frac{\partial}{\partial y}\left(\frac{x^3}{3} - yx + C(y)\right)}_{=\,0 - x + C'(y)} = y^2 - 2x$$

$\boxed{C'(y) = y^2 - x} \rightarrow \boxed{?}$

$C(y) = \frac{y^3}{3} - \int x(y)\,dy$ $\;\;= \boxed{?}$

$\Rightarrow C'(y) = y^2 \Rightarrow C(y) = \int y^2\,dy = \frac{y^3}{3} + C$

$F(x,y) = \frac{x^3}{3} - xy + \frac{y^3}{3} + C$

ANSWER: $x^3 - 3xy + y^3 = D$

$(D = -3C)$

$$\underbrace{P(x,y)}_{=\frac{\partial F}{\partial x}}\,dx + \underbrace{Q(x,y)}_{=\frac{\partial F}{\partial y}}\,dy = 0$$

$$\boxed{\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}}$$

NECESSARY CONDITION

$$\begin{cases} \frac{\partial F}{\partial x} = P(x,y) \\ \frac{\partial F}{\partial y} = Q(x,y) \end{cases} \Rightarrow \begin{aligned} &\frac{\partial}{\partial y}\left(\frac{\partial F}{\partial x}\right) = \frac{\partial P}{\partial y} \\ &\qquad \| \\ &\frac{\partial}{\partial x}\left(\frac{\partial F}{\partial y}\right) = \frac{\partial Q}{\partial x} \end{aligned}$$

$\boxed{\text{SCHWARZ}} \to \|$

$$\frac{\partial^2 F}{\partial x^2}, \frac{\partial^2 F}{\partial y^2} \qquad \| \qquad \frac{\partial^2 F}{\partial x \partial y}$$

IF $P(x,y) = x^2 - y$
$Q(x,y) = y^2 - ax$

$$\frac{\partial P}{\partial y} = -1$$

$?\|$

$$\frac{\partial Q}{\partial x} = -a$$

EXACT $\Leftrightarrow$ $a = 1$

$$y\,dx + x\,dy = 0 \qquad \underbrace{y}\ \ \underbrace{x}_{Q = N}$$

$\frac{dy}{y} = -\frac{dx}{x}$ $\quad P = M$

EXACTNESS COND. $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$

$\frac{\partial P}{\partial y} = 1$, $\frac{\partial Q}{\partial x} = 1$

$$\begin{cases} \frac{\partial F}{\partial x} = P = y \\ \frac{\partial F}{\partial y} = Q = x \end{cases} \quad \Big| \Rightarrow f(x,y) = \int y\,dx = xy + C(y)$$

$$\frac{\partial}{\partial y}\left[xy + C(y)\right] = \cancel{x}$$

$$= \cancel{x} + C'(y)$$

$$C'(y) = 0 \Rightarrow C(y) = C$$

$$F(x,y) = xy + C$$

ANSWER: $\boxed{xy = C}$ OR $\boxed{y = \frac{C}{x}}$

NOT EXACT $\quad y\,dx + 2x\,dy = 0$

$\frac{dy}{y} = -\frac{dx}{2x} \Rightarrow y = \frac{C}{\sqrt{x}}$

$\frac{\partial P}{\partial y} \neq \frac{\partial Q}{\partial x}$

$\frac{\partial P}{\partial y} = 1$, $\frac{\partial Q}{\partial x} = 2$

INTEGRATING FACTOR $\boxed{\cdot y}$

EXACT $\quad \underbrace{y^2}_{P}\,dx + \underbrace{2xy}_{Q}\,dy = 0$

$$2y = \frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x} = 2y$$

$$\begin{cases} \frac{\partial F}{\partial x} = y^2 \Rightarrow F(x,y) = xy^2 + C(y) \\ \frac{\partial F}{\partial y} = 2xy \end{cases}$$

$$2\cancel{x}y + C'(y) = 2\cancel{x}y$$

$$C'(y) = 0 \Rightarrow C(y) = C$$

$$F(x,y) = xy^2 + C$$

sol: $\boxed{xy^2 = C}$

ORIGINAL EQUATION: $\underbrace{P\,dx + Q\,dy = 0}_{\text{NOT EXACT}}$ $\qquad P'_y = \frac{\partial P}{\partial y} \neq \frac{\partial Q}{\partial x} = Q'_x$

NEW EQUATION: $\underbrace{(\mu P)\,dx + (\mu Q)\,dy = 0}_{\text{(WE WANT IT TO BE) EXACT}}$ $\qquad \underbrace{(\mu P)'_y \overset{(*)}{=} (\mu Q)'_x}$

$$(*) \quad \mu'_y \cdot P + \mu \cdot P'_y = \mu'_x \cdot Q + \mu \cdot Q'_x$$

$$(**) \quad \mu'_x Q - \mu'_y P = \mu\left(P'_y - Q'_x\right)$$

IF $\mu = \mu(x)$ (μ DOESN'T DEPEND ON y)

$$(**) \quad \mu'(x) \cdot Q = \mu(P'_y - Q'_x) \Rightarrow \frac{\mu'(x)}{\mu(x)} = \frac{P'_y - Q'_x}{Q}$$

SHOULD BE INDEPENDENT OF y

IF $\mu = \mu(y)$

$$-\mu'(y) P = \mu(P'_y - Q'_x) \Rightarrow \frac{\mu'(y)}{\mu(y)} = \frac{Q'_x - P'_y}{P}$$

IND OF x

$$y\,dx + 2x\,dy = 0 \qquad P = y \quad P'_y = 1 \qquad Q = 2x \quad Q'_x = 2$$

a) $\dfrac{P'_y - Q'_x}{Q} = -\dfrac{1}{2x}$ $\underline{\underline{\text{IS}}}$ IND. OF y

$$\ln|\mu(x)|' = \frac{\mu'(x)}{\mu(x)} = -\frac{1}{\cancel{2x}} \Rightarrow \quad \ln|\mu(x)| = -\frac{1}{2}\ln x + C \Rightarrow \mu(x) = C x^{-\frac{1}{2}}$$

$$\boxed{\mu(x) = x^{-\frac{1}{2}}}$$

b) $\dfrac{Q'_x - P'_y}{P} = \dfrac{1}{y}$ $\underline{\underline{\text{IS}}}$ IND. OF x

$$\frac{\mu'(y)}{\mu(y)} = \frac{1}{y} \qquad \int \frac{d\mu}{\mu} = \int \frac{dy}{y} \Rightarrow \mu(y) = C \cdot y \qquad \boxed{\mu(y) = y}$$

LINEAR DIFF. EQ. y - UNKNOWN FUNC.

1^{st} ORDER: $\underbrace{p(x)\cdot y'(x)+q(x)\cdot y(x)}_{\text{HE}} = r(x)$ p,q,r - GIVEN FUNC.

2^{nd} ORDER $a_2(x)\,y''(x)+a_1(x)\,y'(x)+a_0(x)\cdot y(x)=b(x)$ GIVENS

UNKNOWN

n^{th} ORDER $a_n(x)\,y^{(n)}(x)+\dots+a_0(x)\cdot y(x)=b(x)$

HOMOGENEOUS EQ. $a_n(x)\,y^{(n)}(x)+\dots+a_0(x)\,y(x)=0$

$\underbrace{y''(x)+p(x)y'(x)+q(x)y(x)=0}$

LINEAR DIFF. EQ. WITH CONSTANT COEFFICIENTS

NO GENERAL METHOD

$a_n y^{(n)}(x)+a_{n-1}y^{(n-1)}(x)+\dots+a_1 y'(x)+a_0 y(x)=b(x)$

Examples 1) $y' - y = 0 \longrightarrow y(x) = Ce^{x}$

2) $a y'(x) + b y(x) = 0 \quad (a \neq 0)$

$$\frac{dy}{dx} = y'(x) = -\frac{b}{a} y(x) \Rightarrow \int \frac{dy}{y} = -\frac{b}{a}\int dx \Rightarrow \ln|y| = -\frac{b}{a}x + D$$

$$y = Ce^{-\frac{b}{a}x}$$

3) $\boxed{y'' - 4y' + 3y = 0}$

$t^2 - 4t + 3 = t^2 - t - 3t + 3 =$
$= (t^2 - t) - 3(t-1) = t(t-1) - 3(t-1)$
$= (t-3)(t-1)$

$$y'' - y' - 3y' + 3y = (\underbrace{y' - y}_{= z(x)})' - 3(\underbrace{y' - y}_{= z(x)})$$

$z(x) = y'(x) - y(x)$

$z' - 3z = 0 \Leftrightarrow z = Ce^{3x}$

$y'(x) - y(x) = Ce^{3x}$

HE: $y'(x) - y(x) = 0 \Leftrightarrow y(x) = De^{x}$

VAR.^{N} OF CONSTANTS: $y(x) = D(x)e^{x}$

$y'(x) = D'e^{x} + De^{x}$

$\rightarrow y' - y = D'e^{x}$

$D'(x)e^{x} = Ce^{3x}$

$D'(x) = Ce^{2x} \Rightarrow D(x) = \frac{C}{2}e^{2x} + F$

FINALLY $y(x) = \left(\frac{C}{2}e^{2x} + F\right)e^{x} = \boxed{Fe^{x} + Ge^{3x}}$

EQ: $a_n y^{(n)}(x) + \dots + a_1 y'(x) + a_0 y(x) = 0 \qquad (*)$

$a_n \neq 0$

$\downarrow$

AUXILIARY VARIABLE

CHARACTERISTIC POLYNOMIAL / EQUATION

$$a_n \boxed{r}^n + \dots + a_1 r + a_0 = 0$$

$$\parallel$$

$$a_n \underbrace{(r - \underline{r_1})(r - \underline{r_2}) \dots (r - \underline{r_n})}_{\text{(COMPLEX) ROOTS}}$$

$y'' + y = 0$

$r^2 + 1 = 0$

$\parallel$

$(r - \underline{i})(r + \underline{i})$

SOL: $y(x) = C_1 e^{ix} + C_2 e^{-ix}$

$$\boxed{\begin{aligned} e^{ix} &= \cos x + i \sin x \\ y(x) &= D_1 \cos x + D_2 \sin x \end{aligned}}$$

$\parallel$

$A \cdot \cos(x + B)$

GENERAL SOLUTION OF $(*)$: $y(x) = C_1 e^{r_1 x} + C_2 e^{r_2 x} + \dots + C_n e^{r_n x}$

EX. $y'' - 4y' + 4y = 0 \quad (*)$

CH. POL. $r^2 - 4r + 4 = 0$

$\parallel$

$(r-2)^2 \qquad r_1 = r_2 = 2$

$y(x) = \boxed{C_1 e^{2x}} + C_2 e^{2x}$ $\quad C = C_1 + C_2$

$\boxed{y(x) = C_1 x e^{2x} + C_2 e^{2x}}$ $\quad$ " $C e^{2x}$

SUBSTITUTE $z(x) = \dfrac{y(x)}{e^{2x}}$

$y = e^{2x} \cdot z$

$y' = 2e^{2x} z + e^{2x} \cdot z' = e^{2x}(2z + z')$

$y'' = 4e^{2x} z + 2e^{2x} z' + 2e^{2x} z' + e^{2x} z'' = e^{2x}(4z + 4z' + z'')$

$\Rightarrow (*) \quad e^{2x}(\cancel{4z} + \cancel{4z'} + z'' - \cancel{8z} - \cancel{4z'} + \cancel{4z}) = 0$

$\parallel$

$e^{2x} z'' \Rightarrow z'' = 0$

$z'' = 0 \Rightarrow z' = C_1 \Rightarrow z = C_1 x + C_2 \Rightarrow y(x) = C_1 x e^{2x} + C_2 e^{2x}$

Ex. $y^{(3)} - 3y'' + 3y' - y = 0$

$r^3 - 3r^2 + 3r - 1 = 0$

$\parallel$

$(r-1)^3$

$y(x) = z(x) \cdot e^x$

$y'(x) = \ldots$

$y''(x) = \ldots$

$y'''(x) = \ldots$

$e^x \cdot z'''(x) = 0$

$\Downarrow$

$z'''(x) = 0$

$\Downarrow$

$z(x) = C_1 x^2 + C_2 x + C_3$

3 functions

$\boxed{x^2 e^x, \; x e^x, \; e^x}$

$\rightarrow \quad y(x) = C_1 x^2 e^x + C_2 x e^x + C_3 e^x$

$e^{ix} = \cos x + i \sin x$

Ex. $y^{(4)} + 2y^{(II)} + y = 0$

$r^4 + 2r^2 + 1 = 0$

$\parallel$

$(r^2+1)^2 = (r-i)^2(r+i)^2$

$r_1 = r_2 = i$

$r_3 = r_4 = -i$

SOL.:

$y(x) = C_1 e^{ix} + C_2 x e^{ix} + C_3 e^{-ix} + C_4 x e^{-ix}$

$y(x) = (Dx + E)\cos x + (Fx + G)\sin x$

HOMOGENEOUS EQUATION

$$y'' - 4y' + 3y = 0 \quad (*)$$

$$r^2 - 4r + 3 = 0$$

$$(r-1)(r-3)$$

$$y(x) = C_1 e^{x} + C_2 e^{3x}$$

IF $C_1 = C_2 = 0$ THEN $y(x) \equiv 0$

GENERAL SOLUTION OF HOMOGENEOUS EQ. (GSHE)

GENERAL NONHOM.

NOM.

PARTICULAR

$$GSNE = GSHE + PSNE$$

GENERAL (NON-HOMOGENEOUS) EQ.

$$y'' - 4y' + 3y = 1 \quad (**)$$

THE 'SIMPLEST' SOLUTION OF $(**)$ IS

$$\boxed{y(x) = \frac{1}{3}}$$

$(y_{1/3}'(x) = 0;\ y''(x) = 0)$

REMARK IF $y_1(x)$ AND $y_2(x)$ ARE TWO SOLUTIONS OF $(**)$ THEN

$$y_1'' - 4y_1' + 3y_1 = 1$$

$$y_2'' - 4y_2' + 3y_2 = 1$$

$$(y_1'' - y_2'') - 4(y_1' - y_2') + 3(y_1 - y_2) = 0$$

$z(x) = y_1(x) - y_2(x)$

$$z'' - 4z' + 3z = 0 \quad (*)$$

$$z(x) = C_1 e^{x} + C_2 e^{3x}$$

$$y(x) = C_1 e^{x} + C_2 e^{3x} + \frac{1}{3} \Leftarrow y_1(x) = C_1 e^{x} + C_2 e^{3x} + y_2(x)$$

$$y'' - 4y' + 3y = 2x + 5$$

PREDIGT $y(x) = Ax + B$

$$y'(x) = A$$

$$y''(x) = 0$$

$$y'' - 4y' + 3y = 0 - 4A + 3Ax + 3B$$

$$= 3A \cdot x + 3B - 4A$$

$$\underline{3A} \cdot x + 3B - 4A = \underline{2}x + 5$$

$$\begin{cases} 3A = 2 \Rightarrow A = \frac{2}{3} \\ 3B - 4A = 5 \Rightarrow B = \frac{4A+5}{3} = \frac{23}{9} \end{cases}$$

$$y(x) = \frac{2x}{3} + \frac{23}{9} \in PSNE$$

$$y(x) = C_1 e^{x} + C_2 e^{3x} + \frac{2x}{3} + \frac{23}{9} \in GSNE$$

$$\boxed{y'' - 4y' + 3y = e^{2x}} \quad (**)$$

PREDICT
$$\begin{cases} y(x) = Ae^{2x} \\ y'(x) = 2Ae^{2x} \\ y''(x) = 4Ae^{2x} \end{cases}$$

$$4Ae^{2x} - 4 \cdot 2Ae^{2x} + 3 \cdot Ae^{2x} = e^{2x}$$

$$(4A - 8A + 3A)e^{2x} = e^{2x}$$

$$-Ae^{2x} = e^{2x} \quad | : e^{2x}$$

$$-A = 1$$

$$A = -1$$

$$y(x) = -e^{2x} \quad \leftarrow \text{PSNE}$$

GSNE
$$\boxed{y(x) = C_1 e^{x} + C_2 e^{3x} - e^{2x}}$$

$$\underline{y'' - 4y' + 3y = e^{\cancel{3x}\,4}}$$

$y(x) = A\cancel{x}e^{3x}$

$y(x) = Ae^{4x}$

PREDICTION (INCORRECT)

$$\begin{cases} y(x) = Ae^{x} \\ y'(x) = Ae^{x} \\ y''(x) = Ax \end{cases}$$

$Ae^{x} - 4Ae^{x} + 3Ae^{x} = e^{x}$

$(A - 4A + 3A)e^{x} = e^{x}$

$0 = e^{x}$ $\boxed{??}$

CORRECT PREDICTION

$$\begin{cases} y(x) = Ae^{x} \cdot x \\ y'(x) = \underline{Axe^{x}} + Ae^{x} \\ y''(x) = \underline{Axe^{x} + Ae^{x}} + Ae^{x} \end{cases}$$

$$\underline{(Axe^{x} + 2Ae^{x}) - 4(Axe^{x} + Ae^{x}) + 3Axe^{x} = e^{x}} \quad | \div e^{x}$$

$\cancel{Ax} + 2A - \cancel{4Ax} - 4A + \cancel{3Ax} = 1$

$-2A = 1$

$A = -\frac{1}{2}$

$y(x) = -\frac{1}{2}xe^{x}$ (PSNG)

GSNG $\quad y(x) = C_1e^{x} + C_2e^{3x} - \frac{1}{2}xe^{x}$