Second order linear equations – homework

1. Find the solution of the differential equation

$$y''(x) - 2y'(x) + y(x) = \frac{2e^x}{(x+1)^3}$$

satisfying the initial conditions $y(0) = y'(0) = 2$.

2. Find the solution of the differential equation

$$y''(x) + y'(x) - 6y(x) = 12e^x - 5e^{2x}$$

satisfying the initial conditions $y(0) = -5$ and $y'(0) = 2$.

3. Find the sequence satisfying for all $n \geqslant 0$ the equation

$$a_{n+2} + a_{n+1} - 6a_n = 12 - 5 \cdot 2^n$$

with the initial conditions $a_0 = -5$ and $a_1 = 2$.

4. Find the solution of the system of differential equations

$$\begin{cases} x'(t) &= 3x(t) - 4y(t) + 1 \\ y'(t) &= 4x(t) - 7y(t) + 10t \end{cases}$$

satisfying the initial conditions $x(0) = 5, y(0) = 2$.