SECOND ORDER LINEAR EQUATIONS – REVISION

1. Find the solution of the differential equations
$$y''(x) - 4y'(x) + 3y(x) = 2e^x - 4e^{2x}.$$
satisfying the initial conditions $y(0) = 0$ and $y'(0) = 3$.

ANSWER: $y(x) = -xe^x - 4e^x + 4e^{2x}$

2. Find the solution of the system of differential equations
$$\begin{cases} x'(t) &= y(t) + 2e^t \\ y'(t) &= x(t) - t \end{cases}$$
satisfying the initial condition $x(0) = y(0) = 1$.

ANSWER: $x(t) = te^t + e^t + t, y(t) = te^t + 1$.

3. Find the solution of the system of differential equations
$$\begin{cases} x'(t) &= 3x(t) - 2y(t) \\ y'(t) &= 2x(t) - 2y(t) + e^t \end{cases}$$
satisfying the initial conditions $x(0) = 1, y(0) = 4$.

ANSWER: $x(t) = 2e^{-t} + e^t - 2e^{2t}, y(t) = 4e^{-t} + e^t - e^{2t}$.

4. Find the general solutions of the following differential equation
$$y''(x) - 2y'(x) + y(x) = \frac{e^x}{\sqrt{x}}$$

ANSWER: $y(x) = C_1e^x + C_2xe^x + \frac{4}{3}x^{3/2}e^x$

$$y''(x) + 2y'(x) + y(x) = \frac{\sqrt{x}}{e^x}$$

ANSWER: $y(x) = C_1e^{-x} + C_2xe^{-x} + \frac{4}{15}x^{5/2}e^{-x}$

$$y''(x) - 2y'(x) + y(x) = \frac{x}{e^x}.$$

ANSWER: $y(x) = C_1e^x + C_2xe^x + \frac{xe^{-x}}{4} + \frac{e^{-x}}{4}$

5. Consider the differential equation

$$y''(x) - 6y'(x) + 8y(x) = 4e^{2x}.$$

Assume that we introduce a new unknown function by substituting $y(x) = z(x) \cdot e^{2x}$ and divide the resulting equation by e^{2x}. Write down the new equation and find its solution (at least one).

Answer: Equation: $z''(x) - 2z'(x) = 4$. Possible solution: $z(x) = -2x$