1. How many elements $n \in \{301, 102, \ldots, 900\}$ satisfy the following statement: 'n is not divisible by 8 and n is divisible by 6 or by 10'?

2. Find the union $\bigcup_{n\in\mathbb{N}} A_n$ and intersection $\bigcap_{n\in\mathbb{N}} A_n$ of the sets
$$A_n = \left\langle \frac{1}{n+1}; 2 + \frac{1}{n^2+2} \right) .$$
Assume that $0 \in \mathbb{N}$ and pay special attention to the endpoints of the appropriate intervals.

3. Prove that the relation R defined on the set $X = \{1, 2, \ldots, 100\}$ by

   aRb if and only if $a^2 + ab + b^2$ is divisible by 3

   is an equivalence relation. Determine its equivalence classes $[1]_R, [6]_R$. How many different equivalence classes does this relation have?

4. Determine for what values of the parameters $a, b \in \mathbb{R}$ the operation $x \ddagger y = 2xy + ax - y + b$ defined for $x, y \in \mathbb{R}$ is commutative and associative. Check whether it has a neutral element.

5. Let $X = \{301, 302, \ldots, 900\}$. For every $n \in X$ we define $f(n)$ as the product of the digits of n: $f(853) = 8 \cdot 5 \cdot 3 = 120, f(333) = 27, f(406) = 0$.
   - *i*) For how many elements $n \in X$ is $f(n) = 0$ true?
   - *ii*) For how many elements $n \in X$ is $f(n) > 5$ true?
   - *iii*) For how many elements $n \in X$ is $f(n)$ divisible by 8?

ANSWERS:

1. $100 + 60 - 20 - (25 + 15 - 5) = 105$
2. $\bigcup_{n\in\mathbb{N}} A_n = \left(0; \frac{5}{2}\right)$, $\bigcap_{n\in\mathbb{N}} A_n = \langle 1; 2 \rangle$
3. Three equivalence classes. $[1]_R = \{1, 4, 7, \ldots, 100\}, [6]_R = \{3, 6, 9, \ldots, 99\}$.
4. $a = -1, b = 1$, neutral element $e = 1$.
5. - *i*) 114
   - *ii*) 483
   - *iii*) (probably) 385