- 1. Check if the following formula is a tautology:  $(p \land (q \lor r)) \Leftrightarrow ((p \lor q) \lor (p \land r))$ . Answer. No, the formula is false for example when p is false, q is true, r is false.
- 2. For how many natural numbers n ∈ {33, 34,...,68} is the following sentence true: 'if n is divisible by 4 or by 5, then n is not greater than 52'?
  Answer. 36-6 = 30. It is false only for numbers at the same time greater than 52, and divisible by 4 or by 5, namely 55, 56, 60, 64, 65, 68.
- 3. Find the union  $\bigcup_{n \in \mathbb{N} \setminus \{0\}} A_n$  and intersection  $\bigcap_{n \in \mathbb{N} \setminus \{0\}} A_n$  of the sets

$$A_n = \left(3 - \frac{1}{n^2 + 1}; 5 - \frac{1}{n + 2}\right).$$

**Answer.** Union =  $\left(\frac{5}{2}; 5\right)$ , intersection =  $\left\langle 3; \frac{14}{3} \right\rangle$ .

4. Check if the following formula is a tautology:  $((p \Rightarrow q) \Rightarrow (r \Rightarrow s)) \Leftrightarrow ((p \Rightarrow r) \Rightarrow (q \Rightarrow s))$ 

**Answer.** No, the formula is false for example when q is true and p, r, s are false.

5. Determine the following sets:

a) 
$$\{x \in \mathbb{R} : (\exists y \in \mathbb{R}) ((y^2 < 1) \land (y^2 + 2y = x^2 + 2x))\}$$
  
b)  $\{x \in \mathbb{R} : (\forall n \in \mathbb{N} - \{0\}) 3n + 1 < nx < 5n + 2\}$ 

**Answers.** a)  $x \in (-3; -1) \cup (-1; 1)$  (hint:  $y^2 + 2y = x^2 + 2x$  is true if either x = y or x + y + 2 = 0) b) (4; 5)

6. How many natural numbers  $n \in \{101, 102, \dots, 1000\}$  satisfy the following statement: 'n is divisible by 4 or by 6 but n is not divisible 7'?

**Answer.** (225 + 150 - 75) - (32 + 21 - 10) = 257, where the numbers count elements of the set  $\{101, 102, ..., 1000\}$  divisible respectively by: 4, 6, 4 and 6, 4 and 7, 6 and 7, 4 and 6 and 7.