1. Consider the set $G = \{1, 3, 7, 9\}$ with the operation $x \circ y$ defined as the last decimal digit of the number 3xy.

i) Compute $(3 \circ 3) \circ (3 \circ 3)$. ANS: 7

- ii) Find the neutral element of \circ . ANS: 7
- *iii*) Find the inverse element 1^{-1} . Ans: 9
- 2. Let X be the set of all integers between 400 and 999 (including 400 and 999 themselves) that do not contain the digit 5.

i) How many elements does X have? ANS: $5 \cdot 9 \cdot 9 = 405$

ii) How many elements of X have three even digits? ANS: $3 \cdot 5 \cdot 5 = 75$

iii) How many elements of X have three different digits? ANS: $5 \cdot 8 \cdot 7 = 280$

3. There are 11 blocks in a bag. They have the following letters on them: A A C C C C T T T R R. We choose 4 blocks from the bag, and as we know there are $\binom{11}{4} = 330$ ways of doing that, ie. 330 four-element subsets. In how many of those 330 sets are there

enough blocks to form the word CART? ANS: $4 \cdot 2 \cdot 2 \cdot 3 = 48$

enough blocks to form the word CAT? Correct ANS: 48+12+36+24=120. The other plausible answer $4 \cdot 2 \cdot 3 \cdot 8 = 192$ is wrong.

4^{*} (questions for extra points) Is the set G with the operation \circ described in Problem 1 above a group? ANS: YES.